Dynamics of Continuous, Discrete and Impulsive Systems
Series **A:** *Mathematical Analysis* **16 (2009)** 891-911

BIFURCATIONS AND CHAOS IN CERTAIN PIECEWISE LINEAR DIFFERENTIAL EQUATIONS

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Abstract. Piecewise linear differential equations of autonomous and nonautonomous types are ubiquitous systems modeling important nonlinear dynamical systems in physics and engineering. In particular, they have considerable relevance in the study of bifurcations and chaos in nonlinear electronics. Typical examples include the Chua's circuit, Murali-Lakshmanan-Chua circuit and the negative conductance forced series LCR circuit. In this article, we present a critical overview of some of these lower dimensional systems and show that they admit a wide variety of dynamical states including fixed points, limit cycles, bifurcations of different types to periodic orbits, quasiperiodic attractors, strange nonchaotic, chaotic and hyperchaotic attractors. The existence of these states is demonstrated using exact solutions and numerical analysis. These structures can also be demonstrated by experiments using electronic circuits. Controlling and synchronization in coupled arrays of such systems are also of great practical relevance. Finally we also discuss the bifurcation and chaos scenario in a typical piecewise linear scalar time delayed differential equation.

Keywords. Piecewise linear, nonlinear electronics, time-delays, bifurcations, chaos.

1 Introduction

Nonlinear ordinary differential equations (ODE's) can be broadly classified into two classes: (i) integrable and (ii) nonintegrable ODE's [14]. Integrable systems are characterized by the existence of sufficient number of analytic integrals of motion so that the general solution can be expressed in terms of single-valued, analytic functions involving a certain number of arbitrary constants equal to the order of the equation. Integrable systems admit regular motion, often characterized by periodic or quasiperiodic orbits. On the other hand, nonintegrable systems admit lesser number of independent integrals of motion and the corresponding solutions cannot be in general expressed in terms of explicit analytic functions. These systems can admit regular motions for certain range of system parameters (often called control parameters). More interestingly, they can also admit solutions which are highly sensitive to initial conditions: nearby trajectories diverge exponentially even though the motion is bounded. Such solutions are called chaotic. Several prominent and not so prominent routes have been identified during the past