Dynamics of Continuous, Discrete and Impulsive Systems
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AN EXACTLY SOLVABLE CHAOTIC DIFFERENTIAL EQUATION

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Abstract. We show that continuous-time chaos can be defined using linear dynamics and represented by an exact analytic solution. A driven linear differential equation is used to define a low-dimensional chaotic set of continuous-time waveforms. A nonlinear differential equation is derived for which these waveforms are exact analytic solutions. This nonlinear system describes a chaotic semiflow with a return map that is a chaotic shift map. An extension to the nonlinear differential equation yields an invertible flow with an exact analytic solution and a return map that is a baker's map. Significantly, these exactly solvable nonlinear systems provide the first examples of which we are aware of chaotic ordinary differential equations possessing an exact symbolic dynamics.

Keywords. chaos, linear synthesis, piecewise constant argument, exactly solvable chaos, symbolic dynamics.

AMS (MOS) subject classification: 34C99, 34K23, 37D45, 70K55, 74H05.

1 Introduction

Over the past several decades we have learned that chaotic dynamics are ubiquitous in physics and nature. To deal with this important dynamical behavior a theory of chaotic dynamics has been developed, and the once astonishing and difficult concepts are now accepted and reasonably well understood. From this theory, certain important concepts have crystallized, and now there is a set of commonly held beliefs that are widely accepted as truth. In particular, chaos is commonly viewed as an inherently nonlinear phenomenon, and its complexity and inherent unpredictability seem to preclude simple analytic solution.

In this paper we examine these two commonly held assumptions and show they are not necessarily true. First, we show that a low-dimensional chaotic set of continuous-time waveforms can be defined using linear differential equations. We note that this construction does not give a chaotic linear system; rather, it yields a family of linear systems with solutions that together form a chaotic set. Second, we identify a nonlinear differential equation for which this chaotic set provides an analytic solution. For this system, we also identify an exact symbolic dynamics and show a return map that is a shift map.