

## STABILITY AND BIFURCATION ANALYSIS ON A RATIO-DEPENDENT PREDATOR-PREY MODEL WITH PREY DISPERSAL AND TIME DELAY

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**Abstract.** A ratio-dependent predator-prey model with prey dispersal and time delay due to the gestation of the predator is investigated. By analyzing the corresponding characteristic equation, the local stability of a positive equilibrium and the existence of Hopf bifurcations are established. Using the normal form theory and the center manifold reduction, explicit formulae are derived to determine the direction and stability of bifurcating periodic solutions. By means of an iteration technique, sufficient conditions are obtained to guarantee the positive equilibrium to be globally attractive. Numerical simulations are carried out to illustrate the main results.

**Keywords.** ratio-dependence, predator-prey model, dispersal, time delay, Hopf bifurcation, stability.

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## 1 Introduction

Recently, the traditional prey-dependent predator-prey models have been challenged by several biologists (see, for example, Arditi and Ginzburg [1], Arditi, Ginzburg and Akcakaya [2], Gutierrez [13]) based on the fact that functional and numerical responses over typical ecological timescales ought to depend on the densities of both prey and predators (most likely and simply on their ratio), especially when predators have to search for food (and therefore have to share or compete for food). Such a functional response is called a ratio-dependent response function. These hypotheses are strongly supported by numerous field and laboratory experiment and observations [1,2,3,15]. Based on the Michaelis-Menten or Holling type II function, Arditi and Ginzburg [1] proposed a ratio-dependent function of the form

$$P\left(\frac{x}{y}\right) = \frac{cx/y}{m + x/y} = \frac{cx}{my + x},$$