Dynamics of Continuous, Discrete and Impulsive Systems
Series **A:** *Mathematical Analysis* **16 (2009)** 687-699
 http://www.watam.org

A NECESSARY AND SUFFICIENT CONDITION FOR THE EXISTENCE OF POSITIVE SOLUTIONS TO SINGULAR SECOND-ORDER THREE-POINT BOUNDARY VALUE PROBLEM

Xinsheng Du[1] and Zengqin Zhao[2]

[1,2]School of Mathematics Sciences
Qufu Normal University, Qufu 273165, Shandong, People's Republic of China
email: [1]duxinsheng@qfnu.edu.cn, [2]zqzhao@qfnu.edu.cn.

Abstract. Suppose

(i) $f(t,u,v)$ is nonnegative, continuous on $(0,1)\times(0,+\infty)\times(0,+\infty)$, and nondecreasing on u and nonincreasing on v;

(ii) there exists a real number $b\in(0,+\infty)$ such that for any $r\in(0,1)$,

$$f(t,u,rv)\le r^{-b}f(t,u,v);$$

(iii) there exists a function $g\colon[1,\infty)\to(0,+\infty),\ g(l)<l$ and $g(l)/l^2$ is integrable on $(1,+\infty)$ such that

$$f(t,lu,v)\le g(l)f(t,u,v),\ \forall(t,u,v)\in(0,1)\times(0,+\infty)\times(0,+\infty).$$

Consider the singular boundary value problem

$$\begin{cases} u''(t)+f(t,u(t),u(t))=0,\ 0<t<1,\\ u(0)=au(\eta),\ u(1)=0. \end{cases} \tag{$*$}$$

Then a necessary and sufficient condition for the equation $(*)$ to have $C[0,1]$ positive solutions is that

$$0<\int_0^1 s(1-s)f(s,1,1)ds<\infty.$$

Our nonlinearity may be singular at $t=0$ (and/or $t=1$) and $v=0$.

Keywords. Second order singular three-point boundary value problems; Positive solution; Lower and upper solution; Maximum principle.

AMS (MOS) subject classification: 34B15; 34B16; 34B18

1 Introduction and the Main Results

In this paper, we shall study the singular boundary value problem

$$-u''(t)=f(t,u(t),u(t)),\ 0<t<1, \tag{1.1}$$

with

$$u(0)=au(\eta),\ u(1)=0, \tag{1.2}$$

where $0<a<1,\ 0<\eta<1$, the nonlinear term $f(t,u,v)$ may be singular at $t=0$ (and/or $t=1$) and $v=0$.