USING CAS WAVELETS FOR NUMERICAL SOLUTION OF VOLTERRA INTEGRAL EQUATIONS OF THE SECOND KIND

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Abstract. In this paper, we present a computational method for solving Volterra integral equations of the second kind which is based on the use of CAS wavelets. The operational matrix of integration (OMI) and the product operation matrix (POM) for CAS wavelets are introduced and used to reduce solving the Volterra integral equation to solving a system of algebraic equations. The results presented here demonstrate the validity and efficiency of the technique.

Keywords. Volterra integral equation of second kind; CAS wavelet; Operational matrix of integration; Product operation matrix.

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1 Introduction

Integral equations have significant applications in various fields of science and engineering. These equations are usually difficult to solve analytically and therefore, it is required to obtain approximate solutions. In recent years, many different orthogonal basis functions have been used to approximate the solution of integral equations \cite{6, 7, 9, 13, 15}. The main advantage of using orthogonal basis is that it reduces the problem into solving a system of algebraic equations. Overall, there are so many different families of orthogonal functions which can be used in this method that, it is sometimes difficult to select the most suitable one. Orthogonal wavelets, as very well localized functions, are considerably useful for solving integral equations \cite{5, 16, 17}. Also, the wavelet technique allows the creation of very fast algorithms when compared with the algorithms ordinarily used.

Many problems in fluid dynamics, electrostatics and plasma physics lead to the solution of linear Volterra integral equations of the second kind, namely

\[ u(x) - \int_0^x K(x, y)u(y)\,dy = f(x), \quad 0 \leq x, y \leq 1, \]

where \( f \in L^2[0, 1] \) and \( K(x, y) \in L^2([0, 1] \times [0, 1]) \) are known functions and \( u \) is the unknown function to be determined.