

## SOLUTIONS OF DISTINCT PHYSICAL STRUCTURES FOR TWO GENERALIZED *KdV* EQUATIONS

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**Abstract.** This paper deals with two generalized *KdV* equations which possess three high order nonlinear terms. A technique based on the reduction of order for solving differential equations and variable transformations is employed to find the bell type, kink type, solitary patterns, peakons and periodic solutions for the equations under several circumstances. It is shown that the exponents of the nonlinear terms, the wave speed of the solutions and the coefficients of the derivatives of the equations play a major role in the qualitative change of the physical structures of the solutions.

**Keywords.** Generalized *KdV* equations; Bell type; Kink type; Peakons; Solitary patterns; Periodic solutions.

**AMS (MOS) subject classification:** 35K55, 35B40.

### 1 Introduction

Many mathematical tools have been employed to find travelling wave solutions for nonlinear partial differential equations with dispersive and dissipative effects. Rosenau and Hyman [1] used the pseudo spectral methods in space and a variable order, variable time-step Adams-Basford-Moulton method in time to study a nonlinear dispersive  $K(m, n)$  equation. Deriving the extended tanh method and homogeneous balance method, Fan and Zhang [2] investigated the generalized  $mKdV$  equation and the generalized  $ZK$  equation. The method presented in [2] was a powerful tool to seek exact solutions of nonlinear equations so that many different types of solitary wave solutions were obtained. Ma and Wu [4] used the time and space integrable decomposition technique to get the solutions for the *KdV*, *MKdV* and diffusion equations. Cherniha [5] gave a complete Q-conditional symmetry theory to obtain the exact solutions of the generalized Fisher, Fitzhugh-Nagumo and Kolmogorov-Petrovskii-Piskunov equations. Wazwaz [6] applied the sine-cosine and the tanh methods to find compactons and solitary wave solutions for a Bousinesq wave equation and its generalized form. In fact, one can find many meaningful and important results for several nonlinear