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BOUNDARY VALUE PROBLEMS FOR DYNAMIC EQUATIONS OF VOLTERRA TYPE ON TIME SCALES

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Abstract. This paper considers boundary value problems on time scales and also discusses inequalities on time scales. We formulate sufficient conditions under which such problems have extremal solutions in a corresponding region bounded by lower and upper solutions.
Keywords. Boundary value problems on time scales, inequalities on time scales, existence of solutions.
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1 Introduction

Stefan Hilger [4] introduced the calculus of measure chains in order to unify continuous and discrete analysis. Major works devoted to the calculus on time scales has been introduced in papers [1,3,4,6].

Throughout this paper, we denote by $\mathbb{T}$ any time scale (nonempty closed subset of the real numbers $\mathbb{R}$). We assume that $0, T \in \mathbb{T}$ and denote $J = [0, \sigma(T)] \subset \mathbb{T}$ being a closed interval. Here σ denotes the forward jump operator $\sigma(t) = \inf\{s \in \mathbb{T} : \ s > t\}$. The graininess function $\mu : \mathbb{T} \to [0, \infty)$ is defined by $\mu(t) = \sigma(t) - t$. Let $C(J, \mathbb{R})$ denote the set of continuous functions $u : J \to \mathbb{R}$.

In this paper, we investigate the following first order integro–differential equation of Volterra type on time scales

$$\begin{cases} x^{\triangle}(t) &= f\left(t, x(t), \displaystyle\int_0^t k(t,s)x(s)\triangle s\right) \equiv (\mathcal{F}x)(t), \quad t \in [0,T], \\ x(0) &= rx(\sigma(T)), \quad r \in [0,1], \end{cases} \tag{1}$$

where $f \in C(J \times \mathbb{R} \times \mathbb{R}, \mathbb{R}),\ k \in C(J \times J, \mathbb{R})$.

The monotone iterative method combined with lower and upper solutions has been effectively used for proving the existence results for dynamic equations on time scales, see for example [2,3,7,8], see also [5]. The existence of solutions for dynamic equations of Volterra type with initial conditions was discussed for example in papers [2,7]. Boundary problem (1) was discussed in [8] but only for $r = 1$. The results of our paper extends and improve the