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ALMOST PERIODIC SOLUTIONS OF THE LINEAR DIFFERENTIAL EQUATION WITH PIECEWISE CONSTANT ARGUMENT

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Abstract. The paper is concerned with the existence and stability of almost periodic solutions of linear systems with piecewise constant argument

$$\frac{dx(t)}{dt} = A(t)x(t) + \sum_{j=-N}^{N} A_j(t)x([t+j]) + f(t). \tag{1}$$

where $t \in R, x \in R^n, [\cdot]$ is the greatest integer function. The Wexler inequality [1]-[4] for the Cauchy's matrix is used. The results can be easily extended for the quasilinear case. A new technique of investigation of equations with piecewise argument, based on an integral representation formula, is proposed.
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1 Introduction and Preliminaries

Let Z, R be sets of all integers and real numbers respectively, $||\cdot||$ be the euclidean norm in $R^n, n \in N$, $C_b(R)$ a set of all uniformly continuous and bounded on R functions, $\mathcal{PC}$ a set of all piecewise continuous, right continuous with discontinuities of the first type at points $t = i, i \in Z$, functions. We introduce in $C_b(R)$ and in $\mathcal{PC}$ the sup-norm $||\phi||_0 = \sup_R ||\phi(t)||$.

For $f \in C_b(R)$ and $\tau \in R$ the translate of f by τ is the function $Q_\tau f = f(t+\tau), t \in R$.

A number $\tau \in R$ is called $\epsilon-$ translation number of a function $f \in C_b(R)$ if $||Q_\tau f - f|| < \epsilon$ for every $t \in R$.

Definition 1.1 A function $f \in C_b(R)$ is called almost periodic if for every $\epsilon \in R, \epsilon > 0$, there exists a respectively dense set of $\epsilon-$ translations of $f(t)$.

Denote the set of all such functions as $\mathcal{AP}(R)$.
The aim of this paper is to investigate the problem of existence and exponential stability of an almost periodic solution of system (1).

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