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SINGULAR IMPULSIVE INTEGRAL EQUATIONS

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Abstract. Existence results are presented for the singular integral equations $y(t) = h(t) + \int_0^t q(s)f(s, y(s)) + \sum_{0<t_k<t} a_k y(t_k),\ t \in [0, 1]$. Here f may be singular at $y = 0$.

Keywords . Singular; Integral equation; Positive solution; Impulse; Fixed point theorem.

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1. Introduction

This paper is devoted to the study of existence of solutions to a certain class of singular integral equation of the form

$$y(t) = h(t) + \int_0^t q(s)f(s, y(s))ds + \sum_{0<t_k<t} a_k y(t_k),\ \text{for } t \in [0, 1]. \qquad (1.1)$$

Here the nonlinearity $f(t, y)$ may be singular at $y = 0$, $0 = t_0 < t_1 < t_2 < \cdots < t_p < t_{p+1} = 1$.

The existence of solutions to singular initial and boundary value problems without impulse effects has been investigated before for example in [1-4,6]. However, very few results are available for the case of singular problems with impulses [5]. In this paper we want to fill in this gap and extend the existence results on the case of singular problems with impulses. Our main goal is to present a new existence result for (1.1). However, some technical details are different from [5].

Define $PC[0, 1] = \{u : [0, 1] \to R, u$ is continuous for $t \neq t_k, u(0^+)$, $u(1^-)$, $u(t_k^+)$, $u(t_k^-)$ exist, and $u(t_k^-) = u(t_k), k = 1, \ldots, p\}$ with the norm $||u|| = \sup_{t\in[0,1]} |u(t)|$.

Let K be convex subset of a normed linear space E, V be an open subset of K and $\bar{V}$ and ∂V be the closure of V in K and the boundary of V in K.

Theorem 1.1(Nonlinear Alternative). *Let $N : \bar{V} \to K$ be a compact map with $p \in V$. Then either*

(i) N has a fixed point in $\bar{V}$; or