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FREQUENCY LOCKING IN TISSULAR COUPLING

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Abstract. We expose a framework, inspired by biological observations, dedicated to modeling complex living systems as coupled systems. In particular, we use this framework to adress a main question in the field of living systems: the synchronization phenomenon. This kind of model, named tissular coupling, is quite general and, using different methods from those usually used in this field of research, we reach global results relative to the frequencies locking problem in both finite and continuous populations.

Keywords. Differential systems, finite coupled systems, infinite coupled systems, synchronization, frequency locking.

1 Introduction

Synchronization is an extremely important and interesting emergent property of complex systems. The first example found in literature goes back to the 17th century with Christiaan Huygens' clocks [10, 1]. Although this example rose from artificial systems, this kind of emergent behavior can be found in natural systems at any scale (from cell to whole ecological systems). Indeed, biology abounds with periodic and synchronized phenomena and Ilya Prigogine's work gave a first general explanation to this matter: such behaviors arise from a dissipative structure generally associated to a nonlinear dynamics [19]. Biological systems are open, they evolve far from thermodynamic equilibrium and are subject to numerous regulating processes, leading to highly nonlinear dynamics. Therefore, periodic behaviors appear (with or without synchronization) at any scale [20]. More generally, life itself is governed by circadian rhythms [8]. Those phenomena are as much attractive as they are often spectacular: from cicada populations that appear spontaneously every ten or thirteen years [9] or networks of heart cells that beat together [16] to huge swarms in which fireflies, gathered in a same tree, flash simultaneously [2]. This synchronization phenomenon occupies a privileged position among emergent collective phenomena and more generally in the field of complex