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STRONG FELLER PROPERTIES FOR CONVEX POTENTIAL

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Abstract. In this paper we prove the strong Feller property for semigroups generated by convex potentials U, both in the case when U is finite and when U is infinite. The main tool is a functional analytic theory of convergence of Dirichlet forms in L^2-spaces with changing reference measures.

Keywords. Kolmogorov operators, Dirichlet forms, invariant measures, m-dissipativity, Mosco convergence.

AMS (MOS) subject classification: 47A07, 47D07, 60J60.

1 Introduction

In this paper we consider elliptic operators of the form

$$\mathcal{N}\varphi = \frac{1}{2}\Delta\varphi - \langle DU, D\varphi\rangle,$$

where U is a *convex* potential, that can be finite or not. Under suitable assumptions on U, the realization of $\mathcal{N}$ in L^2 spaces with respect to invariant measures is m-dissipative so it generates a strongly continuous semigroup of contractions $(P_t)_{t\geq 0}$.
Our purpose here is to prove that this semigroup P_t has a regularizing property, called *strong Feller* property, in the sense that for all $\varphi \in B_b(\Omega)^{(1)}$, $t > 0$ the $L^2(\Omega, \nu)$-class $P_t\varphi$ has a Lipschitz continuous ν-version.
If U is finite, i.e. U takes values in $(-\infty, +\infty)$, and superlinear, the probability measure

$$\nu(dx) = \frac{e^{-2U(x)}}{\int_{\mathbb{R}^d} e^{-2U(y)}\,dy}\,dx = Z\,e^{-2U(x)}\,dx \tag{1}$$

is well-defined and following [4] we can construct an analytic semigroup $(P_t)_{t\geq 0}$ on $L^2(\mathbb{R}^d, \nu)$. Then the main tool to prove the strong Feller property is a functional analytic theory of convergence of Dirichlet forms defined on different Hilbert spaces. The idea is due to K. Kuwae and T. Shioya who developed this framework in [12] as a consequence of research on convergence of

[1] $B_b(\Omega)$ is the space of all real bounded Borel mappings on Ω for any open set $\Omega \subseteq \mathbb{R}^d$.