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MULTIPLE SOLUTIONS OF PERIODIC BOUNDARY VALUE PROBLEMS FOR SECOND ORDER FUNCTIONAL DIFFERENTIAL EQUATIONS WITH IMPULSE

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Abstract. We investigate the existence of multiple positive periodic solutions for a class of second order impulsive functional differential equations. We use fixed-point theory and present two examples to demonstrate the applicability of the main theoretical results.

Keywords. Functional differential equations, impulsive differential equations, periodic boundary value problem, multiple solutions, cone.

AMS (MOS) subject classification: 34B37, 34K45.

1 Introduction

The mathematical modelling of processes with abrupt changes leads impulsive differential equations [8, 15, 33, 38]. This kind of equations also occurs in many applications such as physics, chemical technology, population dynamics, biological systems, economics, and vaccination strategies [1, 6, 7, 10, 18, 34, 35, 37, 39-41]. Now there has been a significant development in impulsive theory, see, for example [9, 16, 21-22,25-27, 31, 32] and references therein.

It should be noted that there are also many papers [2-5, 13, 14, 17, 19, 24, 28, 29, 30, 36] concerned with the solvability of periodic boundary value problems (PBVP, for short) for first or second order functional differential equations in recent years. Some of them even considered PBVP for second order functional differential equation with impulse (see, [2, 3, 20]). For example, in [2], W. Ding and M. Han investigated the following problem

$$\begin{cases} -y''(t) = f(t, y(t), y(w(t))), & t \neq t_k,\ t \in [0, T]; \\ \triangle y\big|_{t=t_k} = I_k(y(t_k)), & \\ \triangle y'\big|_{t=t_k} = I_k^*(y(t_k)), & k = 1, 2, \dots p; \\ y(0) = y(T),\ y'(0) = y'(T), & \\ y(t) = y(0), & t \in [-r, 0). \end{cases}$$