Dynamics of Continuous, Discrete and Impulsive Systems
Series **A:** *Mathematical Analysis* **16 (2009)** 499-516
 http://www.watam.org

COEFFICIENT RECOVERY IN PARABOLIC INITIAL BOUNDARY VALUE PROBLEMS

P.D. DuChateau[1] and T.C. Kull[2]

[1]Department of Mathematics
Colorado State University, Fort Collins, Colorado, US

[2]Department of Mathematics
Winthrop University, Rock Hill, South Carolina, US
Corresponding author email:kullt@winthrop.edu

Abstract. We consider the inverse problem of identifying spatially dependent coefficients in linear, parabolic, partial differential equations with specified initial and boundary conditions. Specifically, we explore the recovery of the diffusivity coefficient in the heat equation using a very limited amount of data on a portion of the spatial boundary. In the process, we show that the coefficient dependent observation operator is injective. With this knowledge, we use the Backus-Gilbert approach, modified with an adjoint method, to construct an approximate solution to the problem. This approach ensures the precision of the resolution is independent of the spatial grids used to approximate the coefficient and the solution to the equation. We show the results of numerical experiments for the single, spatial variable case.

Keywords. inverse problem; partial differential equations; Backus-Gilbert method; adjoint method; identifiability.

AMS (MOS) subject classification: 35R30; 45Q05.

1 Introduction

Determination of physical properties in the interior of a region from indirect measurements taken on the boundary of the region is one of the widely studied inverse problems [2]. Methods that have been developed for this purpose range in degree of complexity from the techniques of electrical impedance tomography (EIT) to procedures based on output least squares [7]. EIT demands a very detailed recovery of the interior properties so as to permit construction of a clear image of objects such as tumors or blood clots [8]. As a result, these methods require very large amounts of measured data and involve considerable computational effort. In theory, the construction of the so called Dirichlet to Neumann map requires knowing output measurements corresponding to every possible input. Of course in practice some reasonable approximation to "all possible inputs" is obtained. In other applications, such as the identification of hydraulic properties of porous media, a less sharp picture of the interior properties is required and so the identification