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MULTIPLE POSITIVE SOLUTIONS FOR A CLASS OF NONHOMOGENEOUS ELLIPTIC EQUATIONS IN ESTEBAN-LIONS DOMAINS WITH HOLES

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Abstract. In this article, we consider the following problem

$$-\Delta u + u = f(x,u) + h(x) \text{ in } \Theta,\ u > 0 \text{ in } \Theta,\ u \in H_0^1(\Theta), \qquad (*)$$

where $0 \leq f(x,u) \leq a_0 u + b_0 u^{p-1}$ for all $x \in \Theta, u \geq 0$ with $a_0 \in [0,1)$, $b_0 > 0$, $2 < p < (2N/(N-2))$, if $N \geq 3$, $2 < p < \infty$ if $N = 2$ and Θ is the upper semi-strip domain with a hole or the upper half space with a hole. We prove that $(*)$ has at least two positive solutions if

$$\|h\|_{H^{-1}(\Theta)} < C_p S(\Theta)^{p/2(p-2)}$$

and $h \geq 0$, $h \not\equiv 0$ in Θ, where $S(\Theta)$ is the best Sobolev constants in $S(\Theta)$ and

$$C_p = b_0^{-1/(p-2)}(p-2)(p-1)^{-(p-1)/(p-2)}(1-a_0)^{(p-1)/(p-2)}.$$

Keywords. Multiple positive solutions, nonhomogeneous, Esteban-Lions domains, the upper semi-strip domain, the upper half space.

AMS (MOS) subject classification: 35J20, 35J25, 35J60.

1 Introduction

Throughout this article, let $N \geq 2$, $2^* = \frac{2N}{N-2}$ for $N \geq 3$, $2^* = \infty$ for $N = 2$, p be a given constant such that $2 < p < 2^*$, and (y,z) be the generic point of $\mathbb{R}^N$ with $y \in \mathbb{R}^{N-1}$, $z \in \mathbb{R}$. Denote by $B^N(x_0;R)$ the $N-$ball, $\mathbb{S}$ the strip domain, $\mathbb{S}^+$ the upper semi-strip domain, $\mathbb{R}^N_+$ the upper half space, Ω the upper semi-strip domain with a hole, $\widetilde{\Omega}$ the upper half space with a hole, Ω_1 the complement in a strip domain of a bounded domain, $\widetilde{\Omega}_1$ the exterior domain as follows:

$B^N(x_0;R) = \{x \in \mathbb{R}^N | |x - x_0| < R\}$,
$\mathbb{S} = \{(y,z) | |y| < r_0\}$,
$\mathbb{S}^+ = \{(y,z) \in \mathbb{S} | z > 0\} \cup B^N(0;r_0)$,
$\mathbb{R}^N_+ = \{(y,z) | z > 0\}$;
$\Omega = \mathbb{S}^+ \setminus \overline{D}$, where $D \subset\subset \mathbb{S}^+$ is a smooth bounded domain in $\mathbb{R}^N$;
$\widetilde{\Omega} = \mathbb{R}^N_+ \setminus \overline{D}$, where $D \subset\subset \mathbb{R}^N_+$ is a smooth bounded domain in $\mathbb{R}^N$
$\Omega_1 = \mathbb{S} \setminus \overline{D}$, where $D \subset\subset \mathbb{S}$ is a smooth bounded domain in $\mathbb{R}^N$;
$\widetilde{\Omega}_1 = \mathbb{R}^N \setminus \overline{D}$, where $D \subset\subset \mathbb{R}^N$ is a smooth bounded domain in $\mathbb{R}^N$,