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DYNAMIC INEQUALITIES AND EQUATIONS OF VOLTERRA TYPE ON TIME SCALES

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Abstract. This paper considers initial value problems on time scales and also discusses inequalities on time scales. Theorem 3 presents an existence result for linear dynamic problems on time scales and we give sufficient conditions for such a problem to have a unique solution. To achieve this we apply a Banach fixed point theorem with a corresponding weighted norm (Bielecki norm).
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1 Introduction

Throughout this paper, we denote by $\mathbb{T}$ any time scale (nonempty closed subset of the real numbers $\mathbb{R}$). By $J = [0, T]$, we denote a subset of $\mathbb{T}$ such that $[0, T] = \{t \in \mathbb{T} : \ 0 \le t \le T\}$. Let $C(J, \mathbb{R})$ denote the set of continuous functions $u : J \to \mathbb{R}$.

In this paper, we investigate the following first order integro–differential equation of Volterra type on time scales

$$\begin{cases} x^{\triangle}(t) &= f\left(t, x(t), \int_0^t k(t,s)x(s)\triangle s\right) \equiv (\mathcal{F}x)(t), \quad t \in J, \\ x(0) &= x_0 \in \mathbb{R}, \end{cases} \tag{1}$$

where $f \in C(J \times \mathbb{R} \times \mathbb{R}, \mathbb{R}), \ k \in C(J \times J, \mathbb{R})$.

Problem (1) was discussed in [7]. The results in our paper improve the corresponding results of [7]. Our first result is a dynamic inequality which we need to show the main result in Section 5. In Theorem 3 we formulate sufficient conditions so that a linear dynamic equation has a unique solution. To obtain such a result we use the Banach fixed point theorem with a corresponding weighted norm. Theorem 4 discusses the existence of extremal solutions of problem (1).