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DISCRETE FREDHOLM OPERATORS, FIXED POINT THEOREMS AND THEIR APPLICATION

Yihong Song[1]

[1]Department of Mathematics
Suzhou University, Suzhou, Jiangsu 215006, P. R. China
E-mail: songyihong@suda.edu.cn

Abstract. The existence of solutions of discrete Fredholm equations is established. We define discrete Fredholm operators on spaces of infinite sequences, and present some of their basic properties. The treatment relies upon the use of coordinate functions, and the existence results are obtained using fixed point theorems for discrete Fredholm operators on infinite-dimensional spaces based on fixed point theorems of Schauder, Rothe, and Altman, for finite-dimensional spaces.

Keywords. Fixed point theorems, Fredholm integral equations, discrete Fredholm operators, discrete Fredholm equations, coordinate mappings.

AMS subject classification: 39A10, 39A12, 39A20, 39A70, 47J25.

1 Introduction

This paper deals with the existence of solutions for discrete Fredholm equations, which are implicit and unbounded order difference equations, by using fixed point theorems for corresponding discrete Fredholm operators. Fredholm integral equations have played an important role in the application of mathematics in various disciplines and have been investigated extensively (see, e.g., [2, 6] and references therein). Discrete Fredholm equations can be regarded as the discrete analogue of Fredholm integral equations, and they arise, indirectly, in the discretization of corresponding Fredholm integral equations. As a result, studying such discrete equations is very important and useful, in particular, for the numerical solutions of Fredholm integral equations.

An example of a discrete Fredholm equation is

$$x(n) = f(n) + \sum_{j=0}^{n+\tau(n)} g(n, j, x(j)), \quad n \geq 0, \tag{1}$$

where n, j are integers, $\tau(n) \geq 0$ is an given integer and depends on n, $x(n)$, $f(n)$, $g(n, j, x(j)) \in \mathbb{R}^d$, and we seek $\{x(n)\}_{n=0}^{\infty}$. Notice that (1) is an implicit and unbounded order difference equation.