Dynamics of Continuous, Discrete and Impulsive Systems
Series **A:** *Mathematical Analysis* **16 (2009)** 413-427

CRITICAL POINT THEORY AND ITS APPLICATION TO A CLASS OF CRITICAL GROWTH ELLIPTIC SYSTEMS

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ABSTRACT: By the critical point theory and the relative category theory we show the existence of at least three solutions for a class of critical growth elliptic system with Dirichlet Boundary condition

$$-\Delta u = au + bv + \frac{2\alpha}{\alpha+\beta} u_+^{\alpha-1} v_+^{\beta} + f \qquad \text{in } \Omega,$$

$$-\Delta v = bu + dv + \frac{2\beta}{\alpha+\beta} u_+^{\alpha} v_+^{\beta-1} + g, \qquad \text{in } \Omega.$$

We first show that the system has a negative solution under suitable conditions on the matrix $A = \begin{pmatrix} a & b \\ b & d \end{pmatrix}$, f, g. We also show by the critical point theory, the linking arguments and the relative category that the system has two more nontrivial solutions under the same conditions on A, f and g.

Keywords. Critical growth elliptic system, relative category, linking argument, critical point theory, eigenvalue of a matrix, boundary value problem.
AMS subject classification: 35J20, 35J50, 35J55.

1 Introduction

In this paper we investigate the multiplicity of solutions for the following class of critical growth elliptic systems with Dirichlet boundary condition

$$\begin{cases} -\Delta u & = au + bv + \frac{2\alpha}{\alpha+\beta} u_+^{\alpha-1} v_+^{\beta} + f \qquad \text{in } \Omega, \\ -\Delta v & = cu + dv + \frac{2\beta}{\alpha+\beta} u_+^{\alpha} v_+^{\beta-1} + g \qquad \text{in } \Omega \\ \quad u & = v = 0 \qquad\qquad \text{on } \partial\Omega, \end{cases} \tag{1.1}$$

where $\Omega \subset R^n$; $\alpha,\ \beta > 1$ are real constants, $\alpha + \beta = 2^*$, where $2^* = \frac{2n}{n-2}$, $n \geq 3$, $u_+ = \max\{u, 0\}$ and $f,\ g \in L^s(\Omega)$ for some $s > n$. Let us set the functions f and g as

$$f = s\phi_1 + f_1, \qquad g = t\phi_1 + g_1,$$

where $s,\ t \in R$, $f_1, g_1 \in L^s(\Omega)$, $s > n$, with

$$\int_\Omega f_1\phi_1 = \int_\Omega g_1\phi_1 = 0, \tag{1.2}$$