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SOLUTIONS FOR A SYSTEM OF INITIAL VALUE PROBLEMS OF NONLINEAR FIRST-ORDER IMPLICIT IMPULSIVE DIFFERENTIAL EQUATIONS IN BANACH SPACES[1]

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Abstract. In this paper, by using the Mönch fixed point theorem, we obtain a new existence theorem of solutions for a system of initial value problems of nonlinear first-order implicit impulsive differential equations in Banach spaces under some suitable conditions.
Keywords. Implicit impulsive differential equation, nonlinear differential equation, initial value problem, fixed point, measure of noncompactness, existence.
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1 Introduction

In this paper, we study the following systems of initial value problems of nonlinear first-order implicit impulsive differential equations in Banach spaces $\mathbb{B}_1$ and $\mathbb{B}_2$: Find $(x, y) : J \times J \to \mathbb{B}_1 \times \mathbb{B}_2$ such that

$$\begin{cases} x'(t) = f(t, x(t), y(t), \lambda_1 x'(t)), \ t \neq t_k, \\ y'(t) = g(t, x(t), y(t), \lambda_2 y'(t)), \ t \neq t_k, \\ \triangle x|_{t=t_k} = I_k(x(t_k)), \ (k = 1, 2, \cdots, m), \\ \triangle y|_{t=t_k} = \hat{I}_k(y(t_k)), \ (k = 1, 2, \cdots, m), \\ x(t_0) = x_0, \quad y(t_0) = y_0, \end{cases} \tag{1}$$

where $J = [t_0, t_0 + a] \subset R = (-\infty, +\infty)$ is a compact interval, $t_0 < t_1 < \cdots < t_m < t_0 + a < +\infty$, $f : J \times \mathbb{B}_1 \times \mathbb{B}_2 \times \mathbb{B}_1 \to \mathbb{B}_1$ and $g : J \times \mathbb{B}_1 \times \mathbb{B}_2 \times \mathbb{B}_2 \to \mathbb{B}_2$ are continuous, $\lambda_1, \lambda_2 \geq 0$ are two constants, $x_0 \in \mathbb{B}_1$, $y_0 \in \mathbb{B}_2$ and for $k = 1, 2, \cdots, m$, $I_k \in C[\mathbb{B}_1, \mathbb{B}_1]$, $\hat{I}_k \in C[\mathbb{B}_2, \mathbb{B}_2]$, $\triangle x|_{t=t_k}$ denotes the jump of

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