

SOLUTIONS OF NONLINEAR SECOND ORDER IMPULSIVE INTEGRO-DIFFERENTIAL EQUATIONS OF MIXED TYPE IN BANACH SPACES¹

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Abstract. By establishing comparison results and a fixed point theorem under a weaker compactness-type condition, the existence theorems of initial value problems are obtained for nonlinear second order impulsive integro-differential equations of mixed type in Banach spaces, which improve some related results.

Keywords. Banach space; Impulsive integro-differential equation; Initial value problems; Measure of noncompactness; Fixed point theorem.

AMS (MOS) subject classification: 45N05, 47H07.

1 Introduction

The theory of impulsive differential equations has been emerging as an important area of investigation in recent years. In paper [1, 2], D. Guo and X. Liu established some existence of solutions to initial value problems for nonlinear first order impulsive integro-differential equations in Banach spaces. In paper [3], using Tonelli's method, F.Chen obtained some existence theorems of extremal solutions for nonlinear impulsive Volterra integral equations with infinite moments of impulse effect on the half line R^+ in Banach spaces. Motivated by [1 – 7], we discuss in this paper the existence of solutions to initial value problems for nonlinear second order impulsive integro-differential of mixed type in Banach spaces. The main results obtained here improve the corresponding results of [6]. Finally, an example is worked out.

We consider the initial value problem (IVP) for nonlinear second order impulsive integro-differential equations of mixed type in a Banach space E :

$$\begin{cases} x'' = f(t, x, Tx, Sx), & t \in J, t \neq t_k, \\ \Delta x|_{t=t_k} = I_k(x(t_k), x'(t_k)), & k = 1, 2, \dots, m, \\ \Delta x'|_{t=t_k} = \bar{I}_k(x(t_k), x'(t_k)), & k = 1, 2, \dots, m, \\ x(0) = x_0, x'(0) = x_1, \end{cases} \quad (1)$$

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