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SOME RESULTS FOR LINEAR IMPULSIVE DELAY DIFFERENTIAL EQUATIONS

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Abstract. Essentially, two main results are given. Firstly, it is shown that the limit of the solution of $(1)-(2)$ is finite as $t \to \infty$ (Theorem 1). Secondly, this limit is formulated in terms of the initial function and the matrix solution of an integral equation (Theorem 2).

Keywords. Asymptotic constancy, impulsive delay differential equation, nonhomogeneous linear differential equation.

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1 Introduction

In this paper, we shall consider the nonhomogeneous linear delay differential equation

$$\left\{ \begin{array}{l} x'(t) = A(t)[x(t) - x(t-\tau)] + f(t), \ t \geq t_0, \ t \neq \theta_i, \\ \Delta x(\theta_i) = B_i[x(\theta_i) - x(\theta_{i-m})] + D_i, \ i \in Z^+ = \{1, \ 2, ...\}, \end{array} \right. \tag{1}$$

where $t_0 \geq 0, \ \theta_i = t_0 + i\tau$ for $i \in Z^+; \ \tau > 0; \ m \in Z^+; \ \theta_{i-m} \in [t_0 - \tau, \ t_0)$ for $i \leq m$ and $\theta_j < \theta_{j+1}$ for $j \in \{1-m, \ 2-m, \ ..., -2, \ -1, \ 0\}$; $\Delta x(\theta_i) = x(\theta_i^+) - x(\theta_i^-), \ x(\theta_i^+) = \lim\limits_{t \to \theta_i^+} x(t), \ x(\theta_i^-) = \lim\limits_{t \to \theta_i^-} x(t) = x(\theta_i)$.

Impulsive and delay differential equations are observed in many fields of science and technology such as biology, engineering and physics. Many dynamic population models are special case of (1). For example, in [11], Cooke and Yorke proposed the scalar equation

$$x'(t) = g(x(t)) - g(x(t-L))$$

as a model for certain population growth if individuals have a constant life span L, where $x(t)$ is a size of the population at time t and $g(x)$ is the birth rate. When $g(x) = ax$ ($a = const.$), then the equation above reduces to Eq.(1) with $A(t) = a, \ \tau = L$, and $f(t) \equiv 0$. Furthermore, the scalar linear delay differential equation of the type of Eq.(1)

$$x'(t) = a(t)(x(t) - x(t-h))$$