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EXISTENCE OF POSITIVE SOLUTIONS FOR SINGULAR BOUNDARY VALUE PROBLEMS FOR 4TH-ORDER IMPULSIVE DIFFERENTIAL EQUATIONS[1]

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Abstract. Using the lower and upper solution technique, we present some necessary and sufficient conditions for the existence of $PC^2([0,1], R_+)$ as well as $PC^3([0,1], R_+)$ positive solutions of singular boundary value problems for fourth-order impulsive differential equations.

Keywords. Singular boundary value problems, impulse, lower and upper solutions, positive solutions.

AMS(MOS)subject classification: 34B37.

1 Introduction

There are many results on the existence of solutions for impulsive differential equations(see[1-2], [5-11], [13-16]) and most of the recent works discussed the first-order and second-order equations (see [1-2], [5],[11], [14-16]). In [6-10], Dajun Guo considered the existence of solutions for nth-order nonlinear impulsive equations. In this paper, we consider problems

$$\begin{cases} x^{(4)}(t) = f(t, x(t), -x''(t)), t \in (0,1), t \neq t_1, \\ \Delta x|_{t=t_1} = I_0(x(t_1)), \\ \Delta x'|_{t=t_1} = I_1(x(t_1)), \\ \Delta x''|_{t=t_1} = -I_2(x(t_1)), \\ \Delta x^{(3)}|_{t=t_1} = -I_3(x(t_1)), \\ x(0) = x(1) = 0, ax''(0) - bx^{(3)}(0) = 0, cx''(1) + dx^{(3)}(1) = 0. \end{cases} \tag{1.1}$$

Different from [6-10], $f(t, x, y)$ may be singular at $t = 0$, $x = 0$ and $y = 0$.

Assume following conditions hold through the paper.

(H_1) $a \geq 0$, $b \geq 0$, $c \geq 0$, $d \geq 0$, $a + b > 0$, $c + d > 0$, $\rho = ac + ad + bc > 0$;

(H_2) $f \in C((0,1) \times (0,\infty) \times (0,\infty), [0,\infty)), f(t, t(1-t), 1) \neq 0, t \in (0,1)$ and there exist constants $\lambda_1, \lambda_2, \mu_1, \mu_2$, $(-\infty < \lambda_i \leq 0 \leq \mu_i, i = 1, 2, \mu_1 +$

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