Dynamics of Continuous, Discrete and Impulsive Systems
Series **B:** *Applications & Algorithms* **16 (2009)** 281-301

ULTIMATE BOUNDEDNESS OF STOCHASTIC FUNCTIONAL KOLMOGOROV-TYPE SYSTEMS

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Abstract. In general, population systems are often subject to environmental noise. To examine effect of the noise on population systems, this paper perturbs the functional Kolmogorov-type system

$$\dot{x}(t) = \mathrm{diag}(x_1(t), \cdots, x_n(t))f(x_t)$$

into the stochastic functional differential equation

$$dx(t) = \mathrm{diag}(x_1(t), \cdots, x_n(t))[f(x_t)dt + g(x_t)dw(t)].$$

This paper studies the stochastic ultimate boundedness, which implies that the population system will be ultimately bounded with large probability. This property is natural from the biological point of view. As the special cases, we discuss some stochastic Lotka-Volterra systems.

Keywords. Stochastic functional differential equations, Kolmogorov-type system, Lotka-Volterra system, stochastically ultimate boundedness, pth moment boundedness.

AMS subject classification: 34K50, 60H10, 92D25, 93E03.

1 Introduction

Population systems are often subject to environmental noise. It is therefore critical in ecology to discuss whether the presence of such noise affects population systems significantly. Recently, stochastic Lotka-Volterra systems receive the increasing attention. [1, 5] reveal that the noise plays an important role to suppress the growth of the solution, and [2, 7] show the stochastic systems behave similarly with the deterministic systems under different stochastic perturbations respectively. These indicate clearly that different structures of environmental noise may have different effects on Lotka-Volterra systems.

As the generalized Lotka-Volterra system, the n-dimensional deterministic Kolmogorov-type system for n interacting species is described by the following differential equation

$$\dot{x}(t) = \mathrm{diag}(x_1(t), \ldots, x_n(t))f(x(t)), \tag{1.1}$$

where $x = (x_1, \ldots, x_n)^T$, $\mathrm{diag}(x_1, \ldots, x_n)$ represents the $n \times n$ matrix with all elements zero except those on the diagonal which are $x_1, \ldots, x_n$, $f(x) =$