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OSCILLATION OF SECOND ORDER NONLINEAR IMPULSIVE DELAY DIFFERENTIAL EQUATIONS

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Abstract. Sufficient conditions are obtained for oscillation of solutions to impulsive delay differential equations of the form

$$\begin{aligned} &[r(t)x'(t)]' + a(t)f(x(\tau(t))) = 0, \quad t \neq \theta_k, \\ &\Delta[r(t)x'(t)]|_{t=\theta_k} + b_k h(x(\tau(\theta_k))) = 0, \quad (t \in \mathbb{R}_+,\ k \in \mathbb{N}), \end{aligned}$$

which include superlinear and sublinear equations as special cases. It is shown that the impulsive perturbations greatly affect the oscillation behavior of the solutions.
Keywords. Oscillation, second order, nonlinear, delay, impulsive, differential equation.
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1 Introduction

We are concerned with the oscillation of solutions of impulsive delay differential equations of the form

$$\begin{aligned} &[r(t)x'(t)]' + a(t)f(x(\tau(t))) = 0, \quad t \neq \theta_k, \\ &\Delta[r(t)x'(t)]|_{t=\theta_k} + b_k h(x(\tau(\theta_k))) = 0, \quad (t \in \mathbb{R}_+,\ k \in \mathbb{N}), \end{aligned} \tag{1}$$

where $\mathbb{R}_+ = (0, \infty)$, $\mathbb{N} = \{1, 2, \ldots\}$, and $\Delta[z(t)]|_{t=\theta} := z(\theta^+) - z(\theta^-)$ in which $z(\theta^\mp) := \lim_{t\to\theta^\mp} z(t)$. For convenience we define $z(\theta) := z(\theta^-)$.

The following conditions are assumed to hold without further mention:

(a) $r \in C^1(\mathbb{R}_+)$, $r(t) > 0$; $a \in C(\mathbb{R}_+)$ $a(t) \geq 0$;

(b) $\tau \in C^1(\mathbb{R}_+)$, $\tau(t) \leq t$, $\tau'(t) \geq 0$, $\lim_{t\to\infty} \tau(t) = \infty$;

(c) $f \in C(\mathbb{R}) \cap C^1(\mathbb{R} \setminus \{0\})$; $h \in C(\mathbb{R})$

(d) $\{\theta_k\}$ is a fixed strictly increasing unbounded sequence of positive real numbers; $\{b_k\}$ is a sequence of positive real numbers;

(e) $xf(x) > 0$, $f'(x) \geq 0$, and $xh(x) > 0$ for $x \neq 0$;