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ON THE EXISTENCE OF A LIMITING REGIME IN THE SENSE OF DEMIDOVICH FOR A CERTAIN FIFTH ORDER NONLINEAR DIFFERENTIAL EQUATION

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Abstract. A solution $X(t)$ of the fifth order nonlinear differential equation

$$x^{(v)} + ax^{(iv)} + bx^{'''} + cx^{''} + dx^{'} + h(x) = p(t, x, x^{'}, x^{''}, x^{'''}, x^{(iv)}) \qquad (*)$$

with a, b, c, d positive constants, h and p continuous, is said to be a Demidovich limiting regime if $(X^2 + X^{'2} + X^{''2} + X^{'''2} + X^{(iv)2}) \leq m$ for a finite m and all $t \in$ R, and if every other solution converges to X as $t \to \infty$. In this paper, we give some sufficient conditions in order for all solutions of the equation $(*)$ to converge to a limiting regime under some boundedness restrictions on the incremental ratio $\frac{h(\zeta+\eta)-h(\zeta)}{\eta}$, $\eta \neq 0$, and prove that this limiting regime is periodic or almost periodic in t according as p is periodic or almost periodic in t, uniformly in $x, x^{'}, x^{''}, x^{'''}, x^{(iv)}$.

Keywords. Demidovich's limiting regime, periodic solutions, almost periodic solutions.

AMS (MOS) subject classification: 34C11, 34C25, 34C27.

1 Introduction

In this paper, we shall consider the fifth order nonlinear differential equation

$$x^{(v)} + ax^{(iv)} + bx^{'''} + cx^{''} + dx^{'} + h(x) = p(t, x, x^{'}, x^{''}, x^{'''}, x^{(iv)}), \qquad (1.1)$$

in which a, b, c, d are positive constants and the functions h and p are assumed continuous. Furthermore the function h is assumed not to be necessary differentiable but only required to satisfy the incrementary ratio

$$\frac{h(\zeta + \eta) - h(\zeta)}{\eta} \in I_0, \quad \eta \neq 0, \qquad (1.1)^{'}$$

where I_0 is a certain sub-interval of the Routh-Hurwitz interval. The function $p(t, x, x^{'}, x^{''}, x^{'''}, x^{(iv)})$ is assumed to have the form

$$p(t, x, x^{'}, x^{''}, x^{'''}, x^{(iv)}) = q(t) + r(t, x, x^{'}, x^{''}, x^{'''}, x^{(iv)})$$