Dynamics of Continuous, Discrete and Impulsive Systems
Series **A:** Mathematical Analysis **16 (2009)** 155-174
 http://www.watam.org

ASYMPTOTIC BEHAVIOR OF NONOSCILLATORY SOLUTIONS OF FOURTH ORDER NONLINEAR DIFFERENCE EQUATIONS

Ravi P. Agarwal[1], Jelena V. Manojlović[2]

[1] Department of Mathematical Sciences
Florida Institute of Technology, Melbourne, FL 32901, U.S.A.
E-mail: agarwal@fit.edu
[2] University of Niš, Faculty of Science and Mathematics
Department of Mathematics and Computer Science
Višegradska 33, 18000 Niš, Serbia and Montenegro
E-mail: jelenam@pmf.ni.ac.yu

Abstract. IWe consider a class of fourth–order nonlinear difference equations of the form

$$(\Omega) \qquad \Delta^2(p_n(\Delta^2 y_n)^\alpha) + q_n y_{n+3}^\beta = 0, \quad n \in \mathbb{N}$$

where α, β are the ratios of odd positive integers, and $\{p_n\}$, $\{q_n\}$ are positive real sequences defined for all $n \in \mathbb{N}(n_0)$ and satisfy the conditions

$$\sum_{n=n_0}^{\infty} \frac{n}{p_n^{1/\alpha}} = \infty \quad \text{and} \quad \sum_{n=n_0}^{\infty} \left(\frac{n}{p_n}\right)^{1/\alpha} = \infty\,.$$

We shall classify the nonoscillatory solutions of (Ω) and establish necessary and sufficient conditions for the existence of nonoscillatory solutions with specific asymptotic behavior.

Keywords. Nonlinear difference equation, nonoscillatory solution, asymptotic behavior, classification of solutions.

AMS (MOS) subject classification: 34C10, 34C15.

1 Introduction

In the last few years, there has been an increasing interest in the study of oscillatory and asymptotic behavior of solutions of difference equations (see monographs [1] and [2], and the references therein). Compared to second-order difference equations, the study of higher-order equations (see [3], [4], [7], [8], [9], [14], [15]) and, in particular, fourth-order difference equations (see [5], [6], [10], [11], [12], [13]) has received considerably less attention.

In this paper we are concerned with the fourth order quasilinear difference equation

$$(\Omega) \qquad \Delta^2(p_n(\Delta^2 y_n)^\alpha) + q_n y_{n+3}^\beta = 0, \quad n \in \mathbb{N}(n_0)$$

where $\mathbb{N}(n_0) = \{n_0, n_0+1, n_0+2, \ldots\}$, n_0 is a positive integer, Δ is the forward difference operator defined by $\Delta y_n = y_{n+1} - y_n$, α, β are the ratios