VISCOSITY APPROXIMATION FOR SOLUTIONS OF FIXED POINTS AND VARIATIONAL SOLUTIONS FOR PSEUDOCONTRACTIONS IN BANACH SPACES

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Abstract. Let $K$ be a nonempty closed convex subset of a Banach space $E$ and $T : K \to K$ be a Lipschitz pseudocontraction. For any fixed Lipschitz strong pseudocontraction $f : K \to K$, it is shown, under appropriate conditions on the sequences of real numbers $\{\alpha_n\}, \{\beta_n\}$ and $\{\gamma_n\}$, that the sequence $\{x_n\}$ defined by the following viscosity approximation: for any fixed $x_1 \in K$,

$$x_{n+1} = \gamma_n x_n + \alpha_n f(x_n) + \beta_n T x_n, \quad \forall n \geq 1,$$

converges strongly to the fixed point of $T$ whenever the path $\{x_t\}$ in $K$ defined by

$$x_t = (1-t)T x_1 + t f(x_1), \quad \forall t \in (0, 1),$$

converges strongly to the fixed point of $T$. If, in particular, $E$ is a reflexive and strictly convex Banach space with a uniformly Gâteaux differentiable norm, then $\{x_n\}$ strongly converges to the fixed point of $T$, which is also the unique solution of the co-variational inequality.

Keywords. Pseudocontractive mappings, viscosity approximations, co-variational inequality, strong convergence.

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1 Introduction

Let $E$ be a real Banach space with dual $E^*$ and $K$ be a nonempty closed convex subset of $E$. Let $J$ denote the normalized duality mapping from $E$ into $2^{E^*}$ given by

$$J(x) = \{f \in E^*, \langle x, f \rangle = \|x\| \|f\|, \|x\| = \|f\|\}, \quad \forall x \in E,$$

where $\langle \cdot, \cdot \rangle$ denotes the generalized duality pairing. We denote $F(T) = \{x \in E : T x = x\}$ by the set of all fixed point for a mapping $T$. 