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VISCOSITY APPROXIMATION FOR SOLUTIONS OF FIXED POINTS AND VARIATIONAL SOLUTIONS FOR PSEUDOCONTRACTIONS IN BANACH SPACES

Yeol Je Cho[1] and Yisheng Song[2]

[1] Department of Mathematics Education and the RINS,
Gyeongsang National University, Chinju 660-701, Korea
E-mail: yjcho@gsnu.ac.kr

[2] College of Mathematics and Information Science,
Henan Normal University, P.R. China, 453007.
E-mail: songyisheng123@yahoo.com.cn

Abstract. Let K be a nonempty closed convex subset of a Banach space E and $T : K \to K$ be a Lipschitz pseudocontraction. For any fixed Lipschitz strong pseudocontraction $f : K \to K$, it is shown, under appropriate conditions on the sequences of real numbers $\{\alpha_n\}, \{\beta_n\}$ and $\{\gamma_n\}$, that the sequence $\{x_n\}$ defined by the following viscosity approximation: for any fixed $x_1 \in K$,

$$x_{n+1} = \gamma_n x_n + \alpha_n f(x_n) + \beta_n T x_n, \quad \forall n \geq 1,$$

converges strongly to the fixed point of T wheneverthe path $\{x_t\}$ in K defined by

$$x_t = (1-t)Tx_t + tf(x_t), \quad \forall t \in (0,1),$$

converges strongly to the fixed point of T. If, in particular, E is a reflexive and strictly convex Banach space with a uniformly Gâteaux differentiable norm, then $\{x_n\}$ strongly converges to the fixed point of T, which is also the unique solution of the co-variational inequality.

Keywords. Pseudocontractive mappings, viscosity approximations, co-variational inequality, strong convergence.

AMS (MOS) subject classification: 47H06, 47J05, 47J25, 47H10, 47H17.

1 Introduction

Let E be a real Banach space with dual $E*$ and K be a nonempty closed convex subset of E. Let J denote the normalized duality mapping from E into 2^{E^*} given by

$$J(x) = \{f \in E^*, \langle x, f\rangle = \|x\|\|f\|, \|x\| = \|f\|\}, \quad \forall\ x \in E,$$

where $\langle \cdot, \cdot \rangle$ denotes the generalized duality pairing. We denote $F(T) = \{x \in E : Tx = x\}$ by the set of all fixed point for a mapping T.