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ON THE ASYMPTOTIC BEHAVIOR OF SOLUTIONS TO SOME NONLINEAR INTEGRAL EQUATIONS OF CONVOLUTION TYPE

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Abstract. For a class of nonlinear integral equations of convolution type we find necessary and sufficient conditions for the boundedness of nonnegative solutions. We also identify conditions for the solution to converge asymptotically to a determined limit.
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1 Introduction

We consider the integral equation

$$u^p(t) = L(t) + \int_0^t P(t-s)\, u(s)\, ds, \qquad t \geq 0,\ p \geq 1, \tag{1}$$

where $L \in C(\mathbb{R}_+, (0,\infty))$, $P \in C(\mathbb{R}_+, \mathbb{R}_+)$, $P \not\equiv 0$. Equation (1) with $p = 2$ and $L(t) \equiv L_0$ occurs in the theory of infiltration of a fluid from a reservoir into an isotropic porous medium, with $u(t)$ representing the height of the fluid at time t, above the horizontal impervious base (see [11]). For this reason, we will be interested in nonnegative solutions to (1).

The existence and uniqueness of solutions to equations of type (1) was considered in several publications [1, 10, 12, 13], while in [5] bounds on solutions were obtained. In this paper we give necessary and sufficient conditions for the boundedness of nonnegative solutions to (1), focusing on the possibility of determining conditions which ensure that a nonnegative solution to (1) has a finite limit as $t \to \infty$. We also determine this limit. Results in this direction were obtained in [6] for special cases of equations of type (1). The upper and lower bounds provided in [5] are not sharp enough to infer the asymptotic behavior of solutions.

In Section 2 we discuss the existence and uniqueness of nonnegative solutions. Section 3 is devoted to the qualitative behaviour of the solutions for $p > 1$, while in Section 4 we point out that the linear case ($p = 1$) differs considerably from the nonlinear cases ($p > 1$).