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NUMERICAL ACCURACY CASE STUDIES OF TWO SYSTEMS WITH INTERMITTENT DYNAMICS: A 2D RIMLESS SPOKED WHEEL AND A 3D PASSIVE-DYNAMIC MODEL OF WALKING

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Abstract. We report in detail on methods used to estimate the accuracy of standard numerical procedures applied to the study of the motions and stability of two systems with discrete dynamics: (1) a 2D (planar) rigid, rimless spoked wheel, or regular polygon, constrained to roll in a vertical plane down a slope and (2) a 3D passive-dynamic walking machine. In the first case, periodic solutions and their stability are known analytically so that both the numerical result and error predictions can be checked. The same numerical and error estimation procedures are used for the second case where no analytical solution is known. The error analysis for the second system permits the claim that the maximum stability eigenvalue modulus associated with a fixed point of a return map is safely below one;*i.e.*, the corresponding periodic motion is asymptotically stable.

Keywords. Walking, dynamics, return map, fixed point, periodic motion, stability eigenvalues, numerical accuracy, Runge-Kutta, Richardson extrapolation, roundoff error, truncation error.

AMS (MOS) subject classification: 65-01, 65G99, 70E18,70E50, 70E60, 70K05, 70K20, 70Q05.

1 Introduction

Using a mathematical model of a two-legged, 3D passive-dynamic (gravity driven, no control) mechanism that exhibited walking-like motions in the lab, Coleman *et al.* [6] carried out numerical calculations that made plausible its apparent periodicity and stability. Because the central claim of that paper regarding stability was based on mathematical numerical results, the authors reported numerical accuracy estimates to support it but without elaborating on the estimation details. Here in this paper we carefully explicate the details of the standard procedures that the authors used to determine the accuracy of the stability calculations for a somewhat non-standard dynamics problem. The purpose of the paper is to perhaps have it serve as a tutorial for others seeking to estimate numerical accuracy in problems with discrete dynamics (especially collisions or impacts) who may be inexpert in the basic computational methods used here.