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GENERALIZED HYERS-ULAM STABILITY OF C^*-TERNARY ALGEBRA HOMOMORPHISMS

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Abstract. Let q be a positive rational number. We prove the generalized Hyers-Ulam stability of homomorphisms in C^*-ternary algebras and of derivations on C^*-ternary algebras for the following Euler-Lagrange type additive mapping:

$$\sum_{i=1}^{n} f\left(\sum_{j=1}^{n} q(x_i - x_j)\right) + nf\left(\sum_{i=1}^{n} qx_i\right) = nq\sum_{i=1}^{n} f(x_i).$$

This is applied to investigate isomorphisms between C^*-ternary algebras.

Keywords. Euler-Lagrange type additive mapping, C^*-ternary algebra isomorphism, generalized Hyers-Ulam stability, C^*-ternary derivation.

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1 Introduction and Preliminaries

A C^*-ternary algebra is a complex Banach space A, equipped with a ternary product $(x, y, z) \mapsto [x, y, z]$ of A^3 into A, which is $\mathbf{C}$-linear in the outer variables, conjugate $\mathbf{C}$-linear in the middle variable, and associative in the sense that $[x, y, [z, w, v]] = [x, [w, z, y], v] = [[x, y, z], w, v]$, and satisfies $\|[x, y, z]\| \leq \|x\| \cdot \|y\| \cdot \|z\|$ and $\|[x, x, x]\| = \|x\|^3$ (see [2, 41]). Every left Hilbert C^*-module is a C^*-ternary algebra via the ternary product $[x, y, z] := \langle x, y \rangle z$.

If a C^*-ternary algebra $(A, [\cdot, \cdot, \cdot])$ has an identity, i.e., an element $e \in A$ such that $x = [x, e, e] = [e, e, x]$ for all $x \in A$, then it is routine to verify that A, endowed with $x \circ y := [x, e, y]$ and $x^* := [e, x, e]$, is a unital C^*-algebra. Conversely, if $(A, \circ)$ is a unital C^*-algebra, then $[x, y, z] := x \circ y^* \circ z$ makes A into a C^*-ternary algebra.

A $\mathbf{C}$-linear mapping $H : A \to B$ is called a *C^*-ternary algebra homomorphism* if

$$H([x, y, z]) = [H(x), H(y), H(z)]$$

for all $x, y, z \in A$. If, in addition, the mapping H is bijective, then the mapping $H : A \to B$ is called a *C^*-ternary algebra isomorphism.* A $\mathbf{C}$-linear mapping $\delta : A \to A$ is called a *C^*-ternary derivation* if

$$\delta([x, y, z]) = [\delta(x), y, z] + [x, \delta(y), z] + [x, y, \delta(z)]$$

for all $x, y, z \in A$ (see [2]).