

ON THE RELATION BETWEEN INTERIOR CRITICAL POINTS OF SOLUTIONS AND PARAMETERS FOR A CLASS OF P-LAPLACIAN BVP WITH NEUMANN-ROBIN CONDITIONS

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Abstract. In this paper, we consider the boundary value problem

$$\begin{cases} -(\varphi_p(u'(x)))' = \lambda f(u(x)), & x \in (0, 1), \\ u'(0) = 0, \\ u'(1) + \alpha u(1) = 0, \end{cases}$$

where $\alpha \in \mathbb{R}^*$, $\lambda > 0$ are parameters, $p > 1$, $\varphi_p(x) := |x|^{p-2}x$ for all $x \in \mathbb{R}^*$, $\varphi_p(0) = 0$ where $(\varphi_p(u'))'$ is the one dimensional p-Laplacian and $f \in C^2[0, \infty)$ such that $f(0) < 0$, and also f is increasing and concave up. We study the relation between λ 's, for which this problem has a solution, and the number of interior critical points of solutions.

Keywords. Semipositone problem, nonnegative solutions, interior critical points, quadrature method.

AMS (MOS) subject classification: 34B15.

1 Introduction

We study the nonlinear two point boundary value problem

$$-(\varphi_p(u'(x)))' = \lambda f(u(x)), \quad x \in (0, 1), \quad (1)$$

$$u'(0) = 0, \quad (2)$$

$$u'(1) + \alpha u(1) = 0, \quad (3)$$

where $\alpha \in \mathbb{R}^*$, $\lambda > 0$ are parameters and $p > 1$, $\varphi_p(x) = |x|^{p-2}x$ for all $x \in \mathbb{R}^*$, $\varphi_p(0) = 0$, $f \in C^2[0, \infty)$ and $f(0) < 0$ (i.e. is the semipositone problem), and we will assume that there exist $\beta, \theta > 0$ such that

- (i) $f(s) < 0$ on $[0, \beta)$,
- (ii) $f(\beta) = 0$,
- (iii) $f'(s) \geq 0$ and $f''(s) > 0$ for $s \geq 0$,
- (iv) $\lim_{s \rightarrow \infty} \frac{f(s)}{|s|^{p-2}s} = \infty$,
- (v) $F(\theta) = 0$ where $F(s) = \int_0^s f(t)dt$.