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CONVECTIVE CAHN-HILLIARD EQUATION WITH DEGENERATE MOBILITY*

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Abstract. In this paper, we study the existence of weak solutions for the convective Cahn-Hilliard equation with degenerate mobility. Based on the Schauder type estimates, we establish the global existence of classical solutions for regularized problems. After establishing some necessary uniform estimates on the approximate solutions, we prove the existence of weak solutions.

Keywords. Convective, Cahn-Hilliard equation, degenerate mobility, existence.

AMS (MOS) subject classification: 35G25, 35K55, 35K65

1 Introduction

In this paper, we investigate the convective Cahn-Hilliard equation

$$\frac{\partial u}{\partial t} + D\left[m(u)(kD^3u - DA(u))\right] - \gamma DB(u) = 0, \quad \text{in} \quad Q_T, \tag{1.1}$$

where $Q_T = (0,1) \times (0,T)$, $D = \frac{\partial}{\partial x}$ and $m(u) = |u|^n, n > 1$, $B(u) = u^2$, and $k > 0, \gamma > 0$ are constants, from the physical consideration, we prefer to consider a typical case of the potential $H(u)$, that is $H'(u) = A(u)$, in the following form

$$(H1) \qquad H(u) = \frac{1}{4}(u^2 - 1)^2,$$

namely, the well known double well potential.

On the basis of physical consideration, as usual the equation (1.1) is supplemented with the natural boundary value conditions

$$u(0,t) = u(1,t) = \frac{\partial^2 u}{\partial x^2}(0,t) = \frac{\partial^2 u}{\partial x^2}(1,t) = 0,\ t > 0. \tag{1.2}$$

The boundary value conditions (1.2) are reasonable for the thin film equation or the Cahn-Hilliard equation (see [1, 2]), and the initial value condition

$$u(x,0) = u_0(x). \tag{1.3}$$

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