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ON THE PLANAR CENTRAL CONFIGURATIONS OF THE 4–BODY PROBLEM WITH THREE EQUAL MASSES

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Abstract. Taking conveniently the unity of mass we can assume that three masses are equal to 1, and the fourth one is m. First, we prove that if a central configuration has an axis of symmetry containing two masses, then one of these two masses is the mass m. Such central configurations will be called kite central configurations. Our main contribution is to give the exact number of kite central configurations when three masses are equal, and the exact values of the masses when these numbers change.

Keywords. 4-body problem, convex and concave central configurations, resultant of two polynomials.

AMS (MOS) subject classification: 70F10, 70F15.

1 Introduction

The main problem of Celestial Mechanics is the n–body problem; i.e. the description of the motion of n particles of positive masses under their mutual Newtonian gravitational forces. It is well known that this problem is completely solved only when $n = 2$. For $n > 2$ there are only few partial results.

Central configurations are initial positions of the n bodies where the position and the acceleration vector of each particle with respect to the center of mass are proportional, and the constant of proportionality is the same for the n particles. The study of central configurations is a very old problem, there is an extensive literature concerning these solutions. For a classical background, see the sections on central configurations in [22] and [6]. For a modern background one can see [18], [4], [12] and [19]. More recent work can be found in [2], [17], [7] and [23].

One of the main reasons why central configurations are important is that they allow to obtain the unique explicit solutions of the n–body problem known until now, the *homographic solutions* for which the ratios of the mutual distances between the bodies remain constant. This was already pointed