

## PARAMETER IDENTIFICATION FOR A NONLINEAR THERMODYNAMIC SYSTEM OF THE ARCTIC SEA ICE

Wei Lv<sup>1</sup>, Enmin Feng<sup>2</sup> and Zhijun Li<sup>3</sup>

<sup>1</sup>Department of Mathematics, Shanghai University  
Shanghai 200444, China E-mails: lvwei7809@yahoo.com.cn

<sup>2</sup>Department of Applied Mathematics, Dalian University of Technology  
Dalian 116024, China, E-mails: emfeng@dlut.edu.cn

<sup>3</sup>State Key Laboratory of Coastal and Offshore Engineering  
Dalian University of Technology, Dalian 116024, China

E-mail: lizhijun@dlut.edu.cn

**Abstract.** In this paper, based on the G.A.Maykut and N.Untersteiner's model and the observed temperature data of the Arctic sea ice from Aug.28 to Oct.9,2003, the thermodynamic parameters of a nonlinear thermodynamic system of the Arctic sea ice are identified. The model provides a theoretical basis for the associated identification problem of the Arctic sea ice systems. Firstly, the existence and uniqueness of weak solution of the system are established. Subsequently, taking the thermodynamic parameters of snow layer, sea ice layer and ocean mixed layer as the control variable, the minimal error of the temperature obtained from the system and the observed temperature data of sea ice as the performance criterion, we construct an optimal control model with state constraints, and prove the strong continuity of solutions with respect to the parameters and the existence of optimal parameter. Finally necessary conditions for optimality are derived.

**Keywords.** Parameter identification, Nonlinear thermodynamic system, Optimal control, Necessary condition for optimality, Arctic sea ice.

**AMS (MOS) subject classification:** 93B30, 35K05, 49K20.

## 1 Introduction

The temperature distribution of sea ice is an important factor in the climate system[1]. Nonlinear thermodynamic system are a suitable mathematical tool to simulate and forecast the temperature distribution of sea ice. The first nonlinear thermodynamic model of sea ice was best described by G.A. Maykut and N. Untersteiner[2](hereinafer referred to as MU). Using a one-dimensional thermodynamic model of sea ice, MU modelled the time-dependent vertical diffusion process occurring within the ice, using prescribed heat fluxes at the atmosphere and oceanic boundaries. Subsequently, many researchers have studied the sea ice thermodynamic models[3-11]. In the thermodynamic system, the density, the specific heat and the thermal conductivity are important components of the physical parameters affecting