

APPROXIMATION OF COMMON FIXED POINTS FOR AN IMPLICIT ITERATION WITH ERRORS OF FINITE FAMILY OF NONSELF NONEXPANSIVE MAPPINGS IN BANACH SPACES

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Abstract. In this paper, we shall study the implicit iterative sequence with errors for a finite family of nonself nonexpansive mappings and proved the strong convergence of the sequence to a common fixed point of the family in a uniformly convex Banach space, requiring only one member in the family $\{T_i : C \rightarrow E, i \in I\}$ to be semi-compact. The results presented in this paper generalized and extend the corresponding main results of Reich [12] and Shioji and Takahashi [13], Wittmann [14], Xu and Ori [16] and therein.

Keywords. Uniformly convex Banach space, retraction, semi-compact, nonself nonexpansive mapping, implicit iterative sequence with errors, common fixed point.

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1 Introduction and Preliminaries

Let C be a closed convex subset of a Banach space E and let T be a nonexpansive mapping from C into C , i.e.,

$$\|Tx - Ty\| \leq \|x - y\| \quad (1.1)$$

for all $x, y \in C$. Denote by $F(T)$ the set of all fixed points of T , that is, $F(T) = \{x \in C : Tx = x\}$. Throughout this paper, we always assume that $F(T) \neq \emptyset$.

Recently, Xu and Ori [16] have introduced an implicitly iterative sequence for a finite family of nonexpansive mappings. Let T_1, T_2, \dots, T_N be self mappings of C and suppose that $\mathcal{F} := \bigcap_{i=1}^N F(T_i) \neq \emptyset$, the set of all common fixed points of $T_i, i = 1, 2, \dots, N$. Hereafter, we will denote the index set $\{1, 2, \dots, N\}$ by I . An implicit iterative sequence $\{x_n\}$ for a finite family of nonexpansive mappings is defined as follows, with $\{t_n\}$ a real sequence in