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PERIODIC SOLUTIONS FOR A RESONANT SYSTEM OF COUPLED TELEGRAPH-WAVE EQUATIONS

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Abstract. In this paper the unique solvability of the periodic-Dirichlet problem for nonlinear telegraph-wave equations with resonance is studied. Our methods involve the use of the global inverse function theorem and a Galerkin method.
Key words. telegraph-wave equation, periodic-Dirichlet problem, generalized solution, existence uniqueness.
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1 Introduction

Let $\Omega := [0, 2\pi] \times [0, \pi] \subset \mathbb{R}^2$, $H := [\mathbb{L}^2(\Omega)]^n$, for integers $n \geq 1$, with H a real Hilbert space with the inner product

$$\langle u, v \rangle_H := \int_0^{2\pi} \int_0^{\pi} \langle u(t,x), v(t,x) \rangle dxdt. \tag{1}$$

Here, $\langle \cdot, \cdot \rangle$ denotes the Euclidean inner product in $\mathbb{R}^n$. The norms induced by $\langle \cdot, \cdot \rangle_H$ and $\langle \cdot, \cdot \rangle$ are denoted by $\| \cdot \|_H$ and $| \cdot |$, respectively.

We consider the following system of coupled telegraph-wave equations

$$u_{tt} - u_{xx} + Cu_t - f(t, x, u) = h(t, x), \qquad \forall (t, x) \in \Omega, \tag{2}$$

with the boundary conditions,

$$u(t, 0) = u(t, \pi) = 0, \quad \forall t \in [0, 2\pi]; \tag{3}$$

$$u(0, x) = u(2\pi, x), \quad u_t(0, x) = u_t(2\pi, x), \quad \forall x \in [0, \pi]. \tag{4}$$

Here, the mapping $f : \Omega \times \mathbb{R}^n \to \mathbb{R}^n$ satisfies the Caratheodory conditions, $h : \Omega \to \mathbb{R}^n$, and C is a $n \times n$ symmetric matrix.

By a generalized solution to the periodic-Dirichlet problem for (2), (3), (4) (**GPDS** for (2), (3), (4), for short) we mean a function $u \in H$ such that

$$\langle u, v_{tt} - v_{xx} \rangle_H + \langle u, Cv_t \rangle_H - \langle f(t, x, u), v \rangle_H = \langle h(t, x), v \rangle_H, \tag{5}$$