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DYNAMICS OF A PREDATOR-PREY MODEL WITH DIFFUSION

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Abstract. In this paper we consider a predator-prey model given by a reaction-diffusion system. It incorporates the Holling-type-II and a modified Leslie-Gower functional responses. We focus on the positive equilibrium global stability, bifurcations and mecanisms responsible for transitions between different kind of dynamics.

Keywords. Predator-prey, Lyapunov function, Bifurcation, Chaos, Self-organization.

AMS (MOS) subject classification: 34, 37, 65

1 Introduction

The dynamics relationships between species and their complex properties are at heart of many important ecological and biological processes. Predator-prey dynamics are a classic and relatively well-studied example of interactions. This paper adresses the analysis of the global stability of the endemic equilibrium, the bifurcations and spatio-temporal dynamics of a system of this type. We assume that only basic qualitative features of the system are known, namely the invasion of a prey population by predators. The local dynamics has been studied in [3, 6]. Similar three dimensional systems with the same functional responses are studied in [2, 8, 9] and in [13, 14] with the delay case. Versions with impulsive term are studied for instance in [19].
This model incorporates the Holling-type-II and a modified Leslie-Gower functional responses. Without diffusion it reads as,

$$\left\{\begin{array}{lcl} \dfrac{dH}{dT} & = & \left(a_1 - b_1 H - \dfrac{c_1 P}{H + k_1}\right) H \\ \\ \dfrac{dP}{dT} & = & \left(a_2 - \dfrac{c_2 P}{H + k_2}\right) P \end{array}\right. \qquad (1)$$

with,

$$H(0) \geq 0\ ,\ \ P(0) \geq 0.$$

H and P represent population densities at time T. r_1, a_1, b_1, k_1, r_2, a_2, and k_2 are model parameters assuming only positive values. a_1 is the growth rate