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SOME FIXED POINT THEOREMS IN HYPER MENGER PROBABILISTIC QUASI-METRIC SPACES

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Abstract. In this paper, we consider complete Hyper Menger probabilistic quasi-metric space and prove some fixed point theorems in this space.
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1 Introduction and Preliminaries

Menger [7] introduced the notion of a probabilistic metric spaces in 1942 and, since then, the theory of probabilistic metric spaces has developed in many directions, especially, in nonlinear analysis and applications [2,5,10]. The idea of Menger was to use distribution functions instead of nonnegative real numbers as values of the metric.

In the sequel, we shall adopt the usual terminology, notation and conventions of the theory of probabilistic Menger metric spaces, as in [2,9,10].

Throughout this paper, the space of all probability distribution functions (briefly, d.f.'s) is denoted by $\Delta^+ = \{F : \mathbb{R} \cup \{-\infty, +\infty\} \longrightarrow [0,1] : F$ is left-continuous and non-decreasing on $\mathbb{R}$, $F(0) = 0$ and $F(+\infty) = 1\}$ and the subset $D^+ \subseteq \Delta^+$ is the set $D^+ = \{F \in \Delta^+ : l^- F(+\infty) = 1\}$. Here $l^- f(x)$ denotes the left limit of the function f at the point x, $l^- f(x) = \lim_{t \to x^-} f(t)$. The space Δ^+ is partially ordered by the usual point-wise ordering of functions, i.e., $F \leq G$ if and only if $F(t) \leq G(t)$ for all t in $\mathbb{R}$. The maximal element for Δ^+ in this order is the d.f. given by

$$\varepsilon_0(t) = \begin{cases} 0, & \text{if } t \leq 0, \\ 1, & \text{if } t > 0. \end{cases}$$