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NONLOCAL PROBLEMS FOR INTEGRODIFFERENTIAL EQUATIONS

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Abstract. This paper is concerned with nonlocal Cauchy problems for integrodifferential equations in Banach spaces. We first extend some properties of semigroups to resolvent operators of integrodifferential equations, so that we are able to obtain new results about the existence of mild solutions for integrodifferential equations, without Lipschitz conditions on nonlinear and nonlocal terms.

Keywords. Nonlocal Cauchy problems, integrodifferential equations, compact resolvent operators and their operator norm continuity, mild solutions.

AMS (MOS) subject classification: 34G, 45K.

1 Introduction

Our objective here is to study nonlocal Cauchy problems for the nonlinear integrodifferential equation

$$u'(t) = Au(t) + \int_0^t B(t-s)u(s)ds + f(t,u(t)), \quad 0 \le t \le T, \tag{1}$$

$$u(0) = u_0 + g(u), \tag{2}$$

in a general Banach space $(X, \|\cdot\|)$, where $u_0 \in X$, and $g : C([0,T], X) \to X$ constitutes a nonlocal Cauchy problem. Eq. (1) is derived from the study of heat conduction in materials with memory (e.g., [5], [8]) and viscoelasticity (e.g., [7]).

Nonlocal Cauchy problems can be applied in physics with better effect than the "classical" Cauchy problem $u(0) = u_0$ since more measurements at $t \ge 0$ are allowed. See, e.g., [2, 3, 9, 13] and references therein for other comments and motivations. We make the following assumption.

(H1). A is a densely defined, closed linear operator in X and generates a C_0-semigroup $T(t)$. Hence $D(A)$ (the domain of A) endowed with the graph