Dynamics of Continuous, Discrete and Impulsive Systems
Series **A:** *Mathematical Analysis* **15 (2008)** 769-814
 http://www.watam.org

MAXIMUM PRINCIPLES AND NONOSCILLATION INTERVALS FOR FIRST ORDER VOLTERRA FUNCTIONAL DIFFERENTIAL EQUATIONS

Alexander Domoshnitsky

Department of Mathematics and Computer Science
Ariel University Center of Samaria, Ariel 44837, Israel

Abstract. Historically many classical questions in the theory of functional differential equations such as nonoscillation, differential inequalities and stability were studied without connection between them. As a result, although assertions about the maximum principles for such equations can be interpreted in a corresponding case as analogs of classical concepts in the theory of ordinary differential equations, they do not imply important corollaries, reached on the basis of this connection between different notions. For example, results associated with the maximum principles in contrast with the cases of ordinary and even partial differential equations do not add so much in problems of existence, uniqueness and comparison of solutions to boundary value problems. One of the main goals of this paper is to build a concept of the maximum principles for functional differential equations through description of the connection between nonoscillation, positivity of Green's functions and stability for these equations. New results in every of these topics are also proposed. New results on existence, uniqueness and stability of nonlinear functional differential equations can be based on the maximum principle results.

Keywords. Functional differential equations, maximum principles, nonoscillation, boundary value problems, Green's functions, Cauchy function, differential inequalities, existence and uniqueness of solutions, exponential stability, generalized periodic problem, integral boundary conditions.

AMS (MOS) subject classification: 34K10, 34K11.

1 Introduction.

In this paper we consider the equation

$$(Mx)(t) \equiv x'(t) + (Bx)(t) = f(t), \quad t \in [0, \omega], \tag{1.1}$$

$$lx = c, \tag{1.2}$$

where $B : C_{[0,\omega]} \to L_{[0,\omega]}$ or $B : C_{[0,\omega]} \to L^{\infty}_{[0,\omega]}$ is a linear continuous Volterra operator, $C_{[0,\omega]}$ is the space of continuous functions, $L_{[0,\omega]}$ is the space of summable and $L^{\infty}_{[0,\omega]}$ is the space of essentially bounded functions