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TOWARD THE RIGOROUS COMPUTATION OF ATTRACTORS AND THEIR BASINS BASED ON SYMBOLIC ANALYSIS

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Abstract. The study of non-linear dynamical systems involves the investigation of their global structures. The localization of attractors and basins are considered as important tasks of such an investigation. In this work, an approach for an efficient numerical computation is proposed which is based on a mathematically proved theoretical concept. As a distinct characteristic, the approach allows the computation of coexisting attractors as well as complex hierarchies of attractors. Furthermore, the domain of attraction can be approximated by an upper and lower bound. The presented algorithms are applied on symbolic images, directed graphs which represent finite discrete approximations of a system flow either discrete or continuous in time.

Keywords. dynamical systems, attractor, domain of attraction, symbolic images, algorithms

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1 Introduction

One of the main objectives of dynamical system theory is to describe the final behavior of some evolving states, i.e. the asymptotic behavior of the states as the time t approaches infinity. The mathematical concept which describes such asymptotic behavior is the attractor. A dynamical system can have a lot of different attractors, and these components can be connected with each other in a complex manner, i.e. there might be hierarchies of coexisting attractors and sequences of attractors embedded into each other. In order to understand the dynamics of a system it is important to reveal all of these attractors and their relationships. Another important consideration is the question by which attractor a trajectory is attracted or, in other words, what is the basin (or domain of attraction) of each attractor. If one can locate all minimal attractors of a dynamical system as well as the basins of these attractors, the long term behavior of each initial state could be predicted. This is especially important for the practical implementation of dynamical system theory. If one knows the attractors and basins of a technical system, proper initial conditions can be set to achieve a desired behavior, or the