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New Concepts for Sequences and Discrete Systems (I)

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Abstract. In this paper, by means of frequency measure (also called asymptotic density or natural density in the earlier literature) defined for subsets of the set of integers, we give a series of new concepts for real sequences and discrete systems and study their properties and simple applications in sequences and discrete systems. These research may clarify several vague mathematical concepts and will form the basis for further research on sequences and discrete systems such as in the fields of pseudo-randomicity of sequences and simulation of random variables, etc.

Keywords. Frequency measure, Frequent distribution, Frequent density, Integral for a sequence.

AMS (MOS) subject classification: 26A03, 28A12, 39A11, 40A05, 46A70.

1 Introduction

A real sequence $X = \{x_k\}_{k=0}^{\infty}$ is said to converge to L if for any $\varepsilon > 0$, there is a real number $N = N_\varepsilon$ such that $|x_n - L| < \varepsilon$ for all $n \geq N$. However, the above definition does not capture the fine details of sequences that do not converge to L. For this reason, definitions such as superior and inferior limits and frequent convergence (or statistical convergence), etc., are introduced. But these definitions are still deficient in the discussion of sequences and discrete systems. This can be seen from the following two sequences $\{\alpha_n\}_{n=1}^{\infty}$ and $\{\beta_n\}_{n=1}^{\infty}$ defined by

$$\alpha_n = \begin{cases} -1/n & n = 10, 10^2, 10^3, \cdots \\ 1 & \text{otherwise,} \end{cases} \tag{1}$$

and

$$\beta_n = \begin{cases} -1/n & n = 1, 3, 5, \cdots \\ 1 & \text{otherwise.} \end{cases} \tag{2}$$

Indeed, although they have the same superior and inferior limits, it appears that the sequence $\{\alpha_n\}$ is near to 1 more 'frequently' than the sequence $\{\beta_n\}$. In other words, almost all terms of $\{\alpha_n\}$ are 1 whereas only half of all terms of $\{\beta_n\}$ are 1. In addition, in view of [5,19,20], we can see that $\{\alpha_n\}$ is frequently convergent (or statistically convergent) to 1 and $\{\beta_n\}$ which is