

MULTIPLE POSITIVE SOLUTIONS TO SINGULAR SECOND ORDER BOUNDARY VALUE PROBLEMS

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Abstract. Under certain conditions, by constructing a special cone and the fixed point theorems of cone expansion and cone compression, we have made a study on a class of second order singular boundary value problems. Some sufficient conditions for the existence of twin $C[0, 1]$ as well as the existence of twin $C_p^1[0, 1]$ positive solutions are obtained.

Keywords. Second order singular differential equation, Decreasing function, Increasing function, Fixed point theorems of cone expansion and cone compression, Positive solution.

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1 Introduction and the Main Results

The singular ordinary differential equations arise in the fields of gas dynamics, Newtonian fluid mechanics, the theory of boundary layer and so on. The existence of positive solution to second order singular boundary value problems has become an important area of investigation in recent years (see [1], [3], [6]-[17] and the references therein). Among these researches, there are studies on superlinear problems (see [3], [7] and [9]-[11]), sublinear problems (see [12] and [14]-[17]), the sum of superlinear problems and sublinear problems (see [6]), and exponent less than zero (see [1] and [13]). In this paper, we will deal with the sum of the three cases mentioned above.

The research on the sum of the three cases mentioned above is rare because it is comparatively difficult. In this paper, we are concerned with the boundary value problems

$$\frac{1}{p(t)}(p(t)u'(t))' + f(t, u(t)) = 0, \quad 0 < t < 1, \quad (1.1)$$

with

$$\alpha u(0) - \lim_{t \rightarrow 0^+} \beta p(t)u'(t) = 0 = \gamma u(1) + \lim_{t \rightarrow 1^-} \delta p(t)u'(t), \quad (1.2)$$

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