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AN APPLICATION OF LIMIT RELATIVE CATEGORY TO THE NONLINEAR HAMILTONIAN SYSTEM WITH POLYNOMIAL INCREASE

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Abstract. We investigate the multiplicity of $2\pi-$periodic solutions of the nonlinear Hamiltonian system with polynomial increase, $\dot{z} = J(H_z(z))$, where $z : R \to R^{2n}$, $\dot{z} = \frac{dz}{dt}$, $J = \begin{pmatrix} 0 & -I \\ I & 0 \end{pmatrix}$, I is the identity matrix on R^n, $H : R^{2n} \to R$, and H_z is the gradient of H. We look for the weak solutions $z = (p, q) \in E$ of the nonlinear Hamiltonian system.

Keywords. Hamiltonian system, Palais-Smale condition, limit relative category, variation linking inequality.

AMS subject classification: 34C25, 34C40, 35B10, 35Q80

1 Introduction

Let $H(z(t))$ be a C^1 function defined on R^{2n} which is $2\pi-$periodic with respect to the variable t. Let $z = (p, q)$, $p = (z_1, \cdots, z_n)$, $q = (z_{n+1}, \cdots, z_{2n})$. In this paper we investigate the multiplicity of $2\pi-$periodic solutions of the following Hamiltonian system with polynomial increase

$$\begin{aligned} \dot{p} &= -H_q(p, q), \\ \dot{q} &= H_p(p, q), \end{aligned}$$

where $H_p(p, q)$ satisfies the below conditions $(H1)$, $(H2)$. The system can be written in a compact version

$$\dot{z} = J(H_z(z)), \tag{1.1}$$

where $z : R \to R^{2n}$, $\dot{z} = \frac{dz}{dt}$, $J = \begin{pmatrix} 0 & -I \\ I & 0 \end{pmatrix}$, I is the identity matrix on R^n, $H : R^{2n} \to R$, and H_z is the gradient of H. Let $E = W^{\frac{1}{2},2}([0, 2\pi], R^{2n})$. We look for the weak solutions $z = (p, q) \in E$ of (1.1); that is, $z = (p, q)$ satisfies

$$\int_0^{2\pi} [(\dot{p} + H_q(z)) \cdot \psi - (\dot{q} - H_p(z)) \cdot \phi] dt$$