

STRONG CONVERGENCE OF THE MODIFIED THREE STEP ITERATIVE PROCESS IN BANACH SPACES*

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Abstract. We study modified three step iterative process for three nonself total asymptotically nonexpansive mappings on a nonempty closed convex subset of a uniformly convex Banach spaces. We derive a necessary and sufficient condition for the strong convergence of this iteration process to a common fixed point of these mappings. The results of this paper improve and extend the corresponding results of [5], [9], [11]–[14], [16], [22] and [26] in the literature.

Keywords. Nonself total asymptotically nonexpansive mappings, modified three step iterative sequence, demicompact, common fixed point, uniformly convex Banach spaces.

AMS (MOS) subject classification: 47H09, 47H10, 47H14

1 Introduction and Preliminaries

We assume that E is a real normed space and K is a nonempty subset of E . A mapping $T : K \rightarrow K$ is called nonexpansive if $\|Tx - Ty\| \leq \|x - y\|$ for all $x, y \in K$. A mapping $T : K \rightarrow K$ is called asymptotically nonexpansive [1] if there exists a sequence $\{k_n\} \subset [1, \infty)$, $\lim_{n \rightarrow \infty} k_n = 1$ such that

$$\|T^n x - T^n y\| \leq k_n \|x - y\| \quad (1.1)$$

for all $x, y \in K$ and $n \geq 1$. Goebel and Kirk [1] proved that if K is a nonempty closed and bounded subset of a uniformly convex Banach space, then every asymptotically nonexpansive self-mapping has a fixed point.

A weaker definition, mapping $T : K \rightarrow K$ is called *asymptotically nonexpansive in the intermediate sense* (see, e.g., [2]) provided that T is continuous and the following inequality holds:

$$\limsup_{n \rightarrow \infty} \sup_{x, y \in K} (\|T^n x - T^n y\| - \|x - y\|) \leq 0. \quad (1.2)$$

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