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MULTIPLICITY RESULTS AND BIFURCATION FOR NONLINEAR ELLIPTIC EIGENVALUE PROBLEMS

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Abstract. In this article, we consider the nonlinear elliptic eigenvalue problems

$$\begin{cases} -\Delta u(x) + u(x) = \lambda(f(x,u) + h(x)) \text{ in } I\!R^N, \\ u \in H^1(I\!R^N),\ u > 0 \text{ in } I\!R^N, \end{cases} \tag{$\star$}$$

where $\lambda > 0$ is a parameter. Under some suitable conditions on f and h, we show that there exists $+\infty > \lambda^* > 0$ such that $(\star)$ has at least two positive solutions for $\lambda \in (0, \lambda^*)$, a unique positive solution u^* for $\lambda = \lambda^*$, some bifurcation results of the solution at $\lambda = \lambda^*$ and further analyzes of the set of positive solutions are made.

Keywords. multiplicity results, bifurcation, nonlinear elliptic eigenvalue problems.

AMS (MOS) subject classification: 35A15, 35J20, 35J25, 35J65.

1 Introduction

We consider the existence, the multiplicity and the bifurcation of positive solutions for the nonlinear elliptic eigenvalue problems

$$\begin{cases} -\Delta u(x) + u(x) = \lambda(f(x,u) + h(x)) \text{ in } I\!R^N, \\ u \in H^1(I\!R^N),\ u > 0 \text{ in } I\!R^N, \end{cases} \tag{$(1.1)_\lambda$}$$

where $\lambda > 0$, $N \geq 3$, $h(x) \in C^\alpha(I\!R^N) \cap L^2(I\!R^N)$, $h(x) \geq 0$, $h(x) \not\equiv 0$, $h(x) \to 0$ as $|x| \to \infty$, and the basic assumptions for the function $f(x,t)$ are

(f1) $f(x,\cdot) \in C^2[0,\infty)$, $f(x,t)$ is measurable in $x \in I\!R^N$, $f(x,t) \equiv 0$ if $t \leq 0$ and $\lim\limits_{t\to 0} \frac{f(x,t)}{t} = 0$ uniformly in $x \in I\!R^N$;

(f2) there exists $C > 0$ such that for all $x \in I\!R^N$ and $t \in I\!R$, $|f(x,t)| \leq C(|t| + |t|^p)$, where $1 < p < \frac{N+2}{N-2}$, $N \geq 3$;

(f3) there exists $\theta \in (0,1)$ such that $\theta t \frac{\partial}{\partial t} f(x,t) \geq f(x,t) > 0$, for all $x \in I\!R^N$, $t > 0$;

(f4) $f(x,t)$ is strictly convex and increasing for $t > 0$;