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OSCILLATION IN SOLUTIONS OF STOCHASTIC DELAY DIFFERENTIAL EQUATIONS WITH RICHARD'S NONLINEARITY

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Abstract. In this paper, we consider a stochastic delay differential equation with Richard's nonlinearity that models the Population Dynamics of a single species. Accordingly, the existence and uniqueness of the positive solutions are established under some conditions. Moreover, the oscillatory behavior of the solutions is studied by means of deterministic theory.

Keywords. Stochastic delay differential equation, oscillation, Richard's nonlinearity, Brownian motion; population dynamics.

AMS (MOS) subject classification: 34K11, 34K50, 34F05

1 Introduction

Delay differential equations (DDE) are widely used in mathematical modeling of Population Dynamics. In recent years, much attention focuses on the solutions of such equations which is oscillatory, as these could plausibly reflect cyclic motion of a system around an equilibrium.

Numerous authors have investigated the oscillation of logistic DDE

$$N'(t) = rN(t)\left(1 - \frac{N(t-\tau)}{C}\right) \tag{1}$$

and its generalized logistic forms (see, e.g. [1]-[7]), where $N(t)$ is the size of a population, or its density, $r \geq 0$ is an intrinsic growth rate, C is a carrying capacity, and $\tau > 0$ is time lag. However, in [8], the logistic nonlinearity is modified to be Richard's nonlinearity, with the form

$$N'(t) = rN(t)\left[1 - \left(\frac{N(t-\tau)}{C}\right)^{\gamma}\right], \tag{2}$$

where $\gamma > 0$. According to [8], $0 < \gamma < 1$ is used for invertebrate population (such as insects, worms, sponges, etc.), and $\gamma \geq 1$ is used for vertebrate populations (such as birds, fish, reptiles, etc.). Some of the results on the oscillation of equation (2) and its generalizations can be found in [8]-[10].