

ALMOST SURE STABILITY OF FUNCTIONAL DIFFERENTIAL EQUATIONS WITH RANDOM IMPULSES¹

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Abstract. By means of Liapunov's direct method coupled with Razumikhin technique, some sufficient conditions for almost sure stability and/or asymptotical stability of the zero solution of functional differential equations with random impulses are presented, where $\frac{dV(t,x(t))}{dt}$ isn't required to be negative definite. Then, the obtained results are applied to the population dynamics to show their applications.

Keywords. Impulsive functional differential equation, almost sure stability, Liapunov function, Razumikhin condition, population dynamics.

AMS (MOS) subject classification: 34A37

1 Introduction

The qualitative properties in the mathematical theory of impulsive differential systems have been very important, of interest and developed by a number of mathematicians, and their studies have attracted much attention [1,2,4]. Furthermore they have been successful in different approaches based on Liapunov's direct method and comparison technique [6,7,9]. In recent years the study of impulsive functional differential equations has been very intensive. Uniform stability of the zero of impulsive functional differential equations has been investigated in [8,10] and uniformly asymptotic stability of the zero of impulsive functional differential equations has been studied in [5,8,10].

For impulsive differential equations, all known literatures deal with them as two kinds of impulsive moments. One is that impulsive moments are a serial of fixed points. The other is that impulsive moments are a serial of functions of "state x ". Thus, impulsive moments are deterministic. However, actual jumps don't always happen at fixed times but usually at random

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