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SUCCESSIVE APPROXIMATIONS TO SOLUTIONS OF SET DIFFERENTIAL EQUATIONS IN BANACH SPACES

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Abstract. By using the method of successive approximations, we prove the existence and uniqueness of a solution for set differential equations with right-hand satisfying a Carathéodory condition in Banach spaces.

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1 Introduction

In the last years, the study of set differential equations in a suitable space was initiated as an independent subject and several basic results on existence, uniqueness, comparison result, global existence and continuous dependence are discussed in many papers ([1], [3], [6], [7], [10]). For results, references and applications in this framework we refer to the book by V. Lakshmikantham, T. Gnana Bhaskar and J. Vasundhara Devi([4]). Also, note that all the concepts not discussed in detail in the sequel can be found in [4]. The aim of this paper is to establish the existence and uniqueness of a solution for set differential equations with right-hand satisfying a Carathéodory condition in Banach spaces using the method of successive approximations that in [11], accordingly adapted.

Let E be a real separable Banach space with norm $\|\cdot\|$. For $x \in E$ and for a closed subset $A \subset E$ we denote by $d(x, A)$ the distance from x to A given by $d(x, A) := \inf\{\|y - x\|; y \in A\}$. For nonempty, bounded closed subsets A, B of E we define the Hausdorff distance between A and B by

$$D[A, B] = \max\{\sup_{x \in B} d(x, A), \sup_{y \in A} d(y, B)\}.$$

Let $K_c(E)$ denote the collection of all nonempty, compact convex subsets of E. Also, we denote by θ zero element of E which is regarded as a one-point set. It is known that $K_c(E)$, endowed with the Hausdorff distance, is a complete separable metric space. Moreover, if the space $K_c(E)$ is equipped with the natural algebraic operations of addition and nonnegative scalar multiplication, then $K_c(E)$ becomes a semilinear metric space which