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IMPULSIVE BOUNDARY VALUE PROBLEMS FOR DYNAMICAL INCLUSIONS ON TIME SCALES

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Abstract. In this paper, the authors investigate the existence of solutions of impulsive boundary value problems for second-order ordinary differential inclusions which admitting non-convex valued on right-hand function. Some new results under weaker conditions are exhibited. The methods rely on a fixed point theorem for contraction multivalued maps due to Covitz and Nadler and Schaefer's fixed point theorem combined with lower semi-continuous multivalued operators with decomposable values.

Keywords. Impulsive differential inclusions, measurable selection, contraction multi-valued map, boundary value problems.

AMS (MOS) subject classification: 34K45, 49K24

1 Introduction

In this paper, we consider the existence of solutions for the following second-order ordinary differential inclusions with the form

$$u''(t) - \lambda u(t) \in F(t, u(t)), \ a.e. \ t \in [0,T] \setminus \{t_1, t_2, \dots, t_m\}, \quad (1.1)$$
$$\Delta u|_{t=t_k} = I_k(u(t_k^-)), \quad k = 1, 2, \dots, m, \quad (1.2)$$
$$\Delta u'|_{t=t_k} = J_k(u(t_k^-)), \quad k = 1, 2, \dots, m, \quad (1.3)$$
$$u(0) - u(T) = \mu_0, \ u'(0) - u'(T) = \mu_1, \quad (1.4)$$

where $F : [0,T] \times R^n \to \mathsf{P}(R^n)$ is a multi-valued map, $I_k, J_k \in C(R^n, R^n)$, $\mu \in R^n$, $\lambda > 0$, $\mathsf{P}(R^n)$ is the family of all nonempty subsets of R^n, $0 = t_0 < t_1 < t_2 < \dots < t_m < t_{m+1} = T$, $\Delta u|_{t=t_k} = u(t_k^+) - u(t_k^-)$, $u(t_k^+)$ and $u(t_k^-)$ represent the right and left limits of $u(t)$ at $t = t_k$, respectively. $\Delta u'|_{t=t_k}$ is defined similarly.

Note that when $\mu_0 = \mu_1 = 0$ we have periodic boundary conditions. When the right hand side is single-valued function, the impulsive ordinary differential equations or inclusions were considered by Nieto[17, 18], Benchohra et al. [3, 4, 5, 6, 7] Bajo and Liz[2] and Pierson Gorez C.[19]. In this