

WEIGHTED EXPONENTIAL TRICHOTOMY OF LINEAR DIFFERENCE EQUATIONS

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Abstract. We introduce the weighted exponential trichotomy notion to difference equations. This kind of trichotomy gives us relevant information about the behavior in the future and the past of solutions for linear systems, both, homogeneous and nonhomogeneous.

Keywords. Difference equation, asymptotic behavior, weighted exponential trichotomy.

AMS (MOS) subject classification: 39A10, 39A11.

1 Introduction

The theory of Difference Equations has received much attention because of its importance in various fields, such as numerical methods of differential equations and dynamical systems, finite elements techniques, control theory, and computer sciences (see for example [1] and [4] and references therein). The notion of dichotomy for a linear system of differential equations has gained prominence since the appearance of two fundamental books: Dalietzkii and Krein [3], and Massera and Schäffer [9]. These were followed by the important book of Coppel [2] who synthesized and improved the results that existed in the literature up to 1978.

Two generalizations of dichotomy in differential equations have been introduced: the first by Sacker and Sell [13], [14], [15] called $(S - S)$ trichotomy and the second by Elaydi and Hájek [7] called $(E - H)$ -trichotomy. In [13] there are given sufficient conditions for the existence of an exponential dichotomy for a class of equations. The problem is treated in the context of linear skew product flows, where it becomes clear how to generalize to the case of fiber preserving flows on vector bundles. In [14] the authors generalize the results obtained in [13], since it includes more cases. Also, sufficient conditions are given guaranteeing the existence of a bounded solution of a linear system. In [15], the study of existence of trichotomy continues and there are included new cases taking into account the eigenvalues of the linear part. In

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